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PSEUDO-BAYESIAN STABILITY OF CSMA AND CSMA/CD LOCAL AREA NETWORKS

by

Murat A. BOYANA

December 1988

Thesis Advisor Tri T. Ha

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Pseudo-Bayesian Stability of CSMA and CSMA/CD Local Area Networks

by

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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

This thesis investigates the stability of the random multiaccess protocols, slotted CSMA and slotted CSMA/CD, utilizing one power level and two power levels to create beneficial power capture effect. Use of more than two equally spaced power levels provides no significant improvement in the throughput achievable when realistic capture thresholds are considered. The investigation centers on a technique known as pseudo-Bayesian stability. Another task of this thesis is to stabilize multichannel slotted CSMA and slotted CSMA/CD with pseudo-Bayesian technique. The multichannel slotted CSMA and slotted CSMA/CD show a large improvement in throughput over a traditional single channel with a combined bit rate.

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I. INTRODUCTION

A. GENERAL BACKGROUND INFORMATION

There are many widely used communication media, such as radio broadcast, satellite systems, multidrop telephone lines, and multitap bus systems, for which the received signal at one node depends on the transmitted signal at two or more other nodes. Typically, such a received signal is the sum of attenuated transmitted signals from a set of other nodes, corrupted by distortion, noise, and delay. Such media, called multiaccess media, form the basis for local area networks (LANS), metropolitan area networks (MANS), satellite networks, and radio networks. Local area networks, which are the subject of this thesis are characterized by restricted physical dimensions. Because of their limited physical extent, the transmission media connecting the nodes of local networks can be installed and owned by the network users. Thus it is economically feasible to use special-purpose, high-speed channels that can operate with very low error rates because of their low noise levels. Typical choices for transmission media are twisted-pair lines, fiber-optic links, 50-ohm coaxial cable (for baseband transmissions) and 75-ohm coaxial cable (for broadband transmissions). The high bit rates that can be used on the node-to-node links enable many users to share the same channel. Often a single shared channel is used for the whole network. Local area networks implement the transfer of data in discrete increments called packets. A packet can range from a few bits in length to several thousand bits, the extremes determined by the particular network. For many of the devices used on local networks, a packet is a natural division of the data.

For local area networks, the random access is a category of multiaccess procedure that regulates the use of the channel by many users and is a key element in the design of local networks. The random access procedures are well matched to the needs of bursty users since the entire bandwidth of a channel can be used by a station once it successfully gains access. Under light load conditions, a user can, on the average, successfully access the channel after only a short waiting period. Other advantages are that no central control is required, stations can be added or removed easily, and most malfunctions are localized to single stations and do not affect the whole network.

Analysis of random multiaccess protocols is made using two parameters: the channel throughput S (defined as the average number of successful transmissions per packet transmission time) and the offered traffic G (defined as the average number of attempted packet transmissions per packet transmission time). A useful performance characteristic for a random multiaccess channel is the relationship between the throughput and offered traffic.

The price to be paid for the advantages of simplicity and short delay times in random access networks is the potential interference between users as they compete for the use of the common channel. Two or more users may decide to transmit at almost the same time so that their signals overlap on the channel, and all are garbled. Such an overlap of signals is called a collision, and the time during which the channel is occupied by corrupted or partially corrupted signals is called a collision or a contention interval. A means for controlling collisions if for the user to sense the presence of other transmissions before attempting transmission; this leads to a new method in a packet radio environment: carrier sense multiple access (CSMA), and its modified implementation carrier sense multiple access with collision detection (CSMA/CD). Efforts to improve the performance leads to the multichannel carrier

sense multiple access (M–CSMA) and multichannel carrier sense multiple access with collision detection (M–CSMA/CD).

B. CSMA, CSMA/CD, M–CSMA AND M–CSMA/CD

To avoid a collision in relatively compact wire or cable networks with stations having short propagation delays, it is possible for a station that has a packet to transmit to listen to the channel to determine if it busy before a transmission is attempted. If the channel is sensed to be busy, the station can defer its transmission until the channel is sensed to be idle. This process is called carrier sensing, and networks that use this process are called carrier sense multiple access (CSMA) networks. The word carrier here, means any electrical activity on the transmission media. Implementation of CSMA protocols requires hardware at each station for sensing the presence of signals on the channel when the station itself is not transmitting. The idea here is to limit the interference among packets by always *rescheduling a packet which finds the channel busy upon arrival*, and *reduce the* number of collisions. [Ref. 2: p. 280]

The advent of local networks initiated an area of application that favors carrier sense procedures and refinements on these procedures. Local networks often use a channel medium that permits a station to monitor the signal on the cable while the station itself is transmitting. This is called "listen while transmitting". If an interfering signal is detected during the monitoring period, the transmission can be aborted immediately. This modification reduces the length of collision intervals since corrupted transmissions can be detected and aborted quickly. The modified operation gives a distinct improvement over CSMA procedure that depend on either the receipt of a positive acknowledgment in a specified time interval after the packet has been transmitted to identify good transmissions, or the lack of such an acknowl-

edgment to determine that a transmission has been corrupted. With the addition of the collision detection feature, the protocol is called carrier sense multiple access with collision detection (CSMA/CD).

The network performances are evaluated considering a variety of multiple access schemes that are derived from CSMA and CSMA/CD protocols by extending the original single channel schemes to a multiple channel system. Multi-channel local area networks (M-LAN's) use a set of lower data rate parallel buses connected to all stations. Buses need not be physically separate; they can be obtained by dividing the bandwidth of a single physical connection. Significant performance improvements can be obtained by using multiple parallel channels. This is due to the fact that with multichannel local area networks, a large bandwidth can be made available to users while employing moderate speed parallel channels. For instance, a 40 Mbit/s total capacity may be provided by five 10 Mbit/s channels, or even by ten 5 Mbit/s connections. The design of each network interface is much simpler in a multichannel local area network than it would be in a local area network with the same capacity using a single broadcast channel. The multichannel local area network architecture is completely modular, so that is allows a gradual system growth following user needs, and also allows an easy network maintenance and repair. Among the advantages introduced with M-LAN's it is important to mention their reliability and fault tolerance. Due to the redundant network structure the disruption of one channel has only a limited impact on the network performance.

C. BASIC MODEL ASSUMPTIONS

The model to be developed allows us to focus on the problem of dealing with the contention that occurs when multiple modes attempt to use the channel simul-

taneously. The system has m transmitting nodes and one receiver. The following assumption are widely used as a basis for the system model:

1. All transmitted packets have the same length and each packet requires one time unit (called a slot) for transmission. All transmitters are synchronized so that the reception of each packet starts at an integer time and ends before the next integer time.

2. The number of users is infinite and the arrival process from this infinite population is Poisson with parameter G. The process includes both newly arrived and previously collided packets. The probability that exactly k packets arrive at the network stations for transmission in an interval of t packet lengths is given by

$$Pr\{k,t\} = \frac{(G \cdot t)^k}{k!} \cdot \exp(-G \cdot t) \tag{1.1}$$

3. If two or more nodes send a packet in a given time slot, then there is a collision and the receiver obtains no information about the contents or source of transmitted packets. Each packet involved in a collision must be retransmitted in some later slot until it is successfully received. A node with a packet that must be retransmitted is said to be backlogged. If just one mode sends a packet in a given slot, then the packet is correctly received.

4. At any point in time, each station has at most one packet ready for transmission, including any previously collided packets.

5. Carrier sensing takes place instantaneously, and there are no turn-around delays in switching from transmitting to receiving.

6. The channel is noiseless so that failure of transmission is due to collisions only.

Relative to the assumptions as a group, the major restrictions result from the facts that the number of users is assumed to be infinite and that no distinction is made between newly generated and retransmitted packets.

For the multichannel local area networks the following additional assumptions are made:

1. The stations are able to detect the presence of a carrier on each one of the M channels simultaneously. This allows the use of CSMA type protocols.

Moreover each interface is able to detect a collision on the channel on which it is transmitting, so that CSMA/CD type protocols can be used. Each station is assumed to be able to transmit on one channel and simultaneously receive from several other channels.

2. A multichannel local network is made of a set of M parallel broadcast channels to which N stations are connected, each one by means of M separate interfaces, one for each channel.

3. All channels have the same bandwidth. Letting W_i be the bandwidth of the ith channel, then the total available bandwidth is

$$W = \sum_{i=1}^{M} W_i \tag{1.2}$$

4. The maximum system throughput is obtained when the available bandwidth is equally divided between the M channels. In this case the optimum probabilities do not depend on the offered traffic, and are equal: [Ref. 7: p. 375]

$$p_i = \frac{1}{M} \tag{1.3}$$

D. PSEUDO-BAYESIAN ALGORITHM

The pseudo-Bayesian algorithm is a particularly simple and effective way to stabilize CSMA and CSMA/CD, and multichannel versions of both. In this algorithm, newly generated packets are regarded as backlogged immediately on arrival at the transmitting station and treated in the same manner as previously collided packets. Rather than being transmitted with certainty in the next slot, they are transmitted with probability q_r in the same way as packets involved in previous collisions. When a slot begins, each active station must decide whether or not to transmit its packet. There are three possible outcomes:

- An idle slot if no station transmits.
- A success slot if one station transmits.
- A collision slot if more than one station transmit and no packets are successfully received.

When an idle or a collision slot occurs, no stations receive any feedback other than the fact that an idle or a collision slot has occurred. All stations are informed of a success slot and the identification of the user that transmitted the successful packet. Each station considers the obtained network idle/success/collision history from this limited feedback to change the estimate of the number of backlogged stations waiting to transmit data and the broadcast probability q_r for subsequent slots. This means that the same value of q_r for each slot will be computed by all network stations. [Ref. 5: pp. 1–2]

The name Bayesian reflects the fact that each station will dynamically estimate the number of active stations on the network using Bayes' Rule. From Bayes' Rule, the probability distribution of the backlogged stations n in the network waiting to transmit data after receiving feedback from the present slot is

$$Pr\{n \mid E\} = \frac{Pr\{E \mid n\} \cdot Pr\{n\}}{Pr\{E\}} \tag{1.4}$$

where E represents the event idle, success or collision of the present slot. Throughout the process, the Poisson assumption on the number of backlogged packets will be used. An approximation to the probability of n, given that a collision occurred in the previous slot, must be made to preserve the Poisson distribution of backlogged packets. For this reason, the algorithm is referred to as pseudo-Bayesian.

The pseudo-Bayesian algorithm is derived by approximating the probability estimates by a Poisson distribution with mean v. Each station keeps a copy of v, transmits a packet with probability $1/v$, and then updates v. The plots are presented demonstrating that the pseudo-Bayesian algorithm, although simple, performs very well.

E. PURPOSE AND OUTLINE

The purpose of this thesis is to analyze the stability of the random multiaccess protocols CSMA and CSMA/CD. Specifically the analysis will center on a recent well-known technique called pseudo-Bayesian stability. The second task is to

stabilize CSMA and CSMA/CD with random two power levels, again using pseudo-Bayesian technique. In addition, the pseudo-Bayesian technique will be used to stabilize multichannel local area networks to achieve performance improvements with the multiple channel option.

Chapter II presents the detailed analysis of slotted (single received power level) CSMA and CSMA/CD using pseudo-Bayesian stabilization. In the same chapter, two power levels for CSMA and CSMA/CD will be considered. Chapter III repeats the theme of Chapter II for slotted multichannel networks with a single power level. Chapter IV presents conclusions and recommendations for further research.

II. PSEUDO-BAYESIAN STABILIZATION OF SLOTTED CSMA AND CSMA/CD

A. STABILIZATION OF SLOTTED CSMA

The pseudo-Bayesian stabilization algorithm assumes that, at the beginning of a slot k, there are n backlogged stations in the network waiting to transmit data including new generated data packets. The value of n is assumed to be a Poisson random variable with mean v, where v represents the user's estimate of the number of backlogged stations in the network. The probability distribution function of n is

$$Pr\{n\} = \frac{v^n \cdot \exp(-v)}{n!} \tag{2.1}$$

Each backlogged packet is independently transmitted with probability q_r, which will vary with the estimated channel backlog v. In state n, the expected number of packets transmitted at the end of an idle slot is

$$G = n \cdot q_r \tag{2.2}$$

The throughput S for CSMA is given by [Ref. 8: p. 1404]

$$S = \frac{G \cdot \exp(-G)}{b + 1 - \exp(-G)} \tag{2.3}$$

where b is the idle slot duration. This ratio can be interpreted as departure rate in state k. Figure 2.1 plots this ratio for $b = 0.01$. The function has a maximum of approximately $1/(1 + \sqrt{2b})$ at $G = \sqrt{2b}$, which is found by maximizing the throughput expression S with respect to G. This can be seen by approximating $\exp(G)$ by $1 + G + G^2/2$.

For small b very little time is wasted on a single idle slot, and significant time is wasted on a collision slot at the point $G = \sqrt{2b}$ where the departure rate Equation 2.3 has its maximum. The departure rate ,eing greater than the arrival rate reduces the backlog . o that the point of operation moves bac.. The point G is where idle slots occur so much more frequently than collision slots that the same expected overall time is wasted on each.

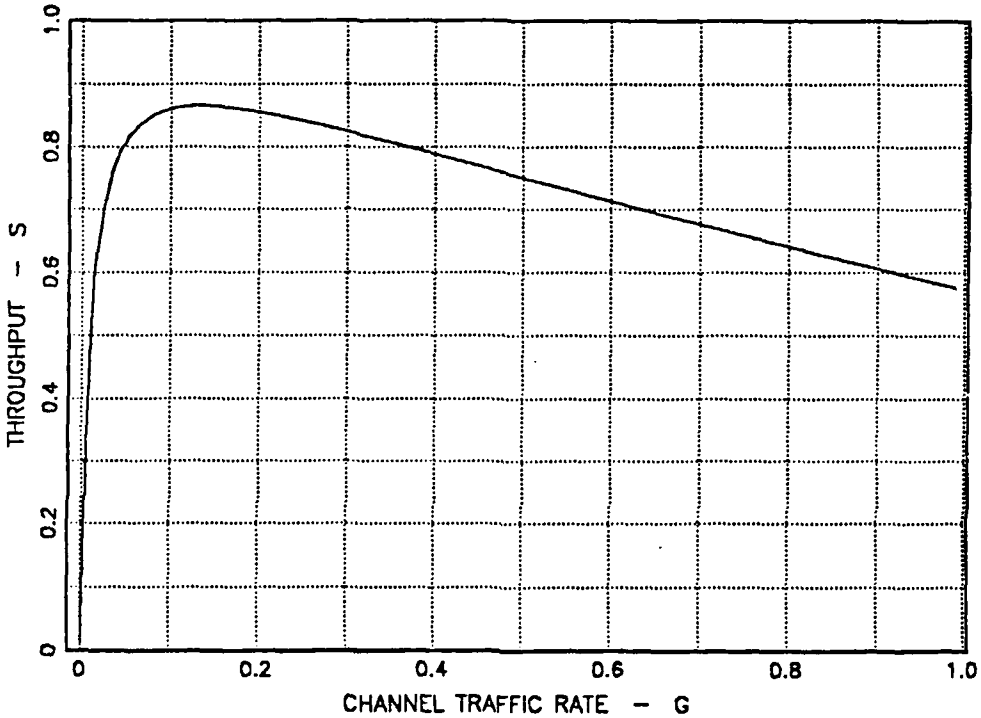

Figure 2.1: The Throughput S for Slotted CSMA as a Function of the Channel Traffic Rate G in Packets per Idle Slot. ($b = 0.01$)

Since it is shown that the packet departure rate (in packets per unit time) is maximized by $G = \sqrt{2b}, q_r$ is chosen for a given estimated backlog v as

$$q_r = \min\left\{\frac{\sqrt{2b}}{v}, \sqrt{2b}\right\} \tag{2.4}$$

The min operation prevents q_r from getting too large when v is small; it cannot be expected for n/v to approach 1 when the backlog is small, and it is desirable to pre-

vent too many collision in this case. The probability that a successful transmission will occur (a success slot S) is the probability that only one of the n backlogged stations transmits while the other $n-1$ users continue to wait. Thus,

$$Pr\{\text{success slot} = S \mid n\} = n \cdot q_r \cdot (1-q_r)^{n-1} \tag{2.5}$$

Averaging over the ensemble of possible values for the number of backlogged users if a successful transmission occurs, the expected probability of a success slot for the optimal broadcast probability q_r given by Equation 2.4 is

$$\begin{aligned} Pr\{S\} &= \sum_{n=1}^{\infty} Pr\{S \mid n\} \cdot Pr\{n\} \\ &= \sum_{n=1}^{\infty} n \cdot q_r \cdot (1-q_r)^{n-1} \cdot \frac{v^n \cdot \exp(-v)}{n!} \\ &= v \cdot q_r \cdot \exp(-v \cdot q_r) \\ &= \sqrt{2b} \cdot \exp(-\sqrt{2b}) \end{aligned} \tag{2.6}$$

Each of the n packets is independently transmitted into slot k with probability q_r, so the probability that the kth slot is idle, given the number n of backlogged stations in the network at a given time, is

$$Pr\{I \mid n\} = (1-q_r)^n = \left(1 - \frac{\sqrt{2b}}{v}\right)^n \tag{2.7}$$

The expected probability of an idle slot is determined by averaging over n; that is

$$\begin{aligned} Pr\{I\} &= \sum_{n=0}^{\infty} Pr\{I \mid n\} \cdot Pr\{n\} \\ &= \sum_{n=0}^{\infty} \left(1 - \frac{\sqrt{2b}}{v}\right)^n \cdot \frac{v^n \cdot \exp(-v)}{n!} \\ &= \exp(-v) \cdot \sum_{n=0}^{\infty} \frac{(v-\sqrt{2b})^n}{n!} = \exp(-\sqrt{2b}) \end{aligned} \tag{2.8}$$

Application of Bayes' Rule in Equation 1.4 yields

$$Pr\{n \mid I\} = \frac{Pr\{I \mid n\} \cdot Pr\{n\}}{Pr\{I\}} = \frac{(v-\sqrt{2b})^n \cdot \exp(-v+\sqrt{2b})}{n!} \tag{2.9}$$

Applying Bayes' Rule to Equation 2.1, 2.5 and 2.6, the probability distribution of n after a success slot is

$$Pr\{n \mid S\} = \frac{Pr\{S \mid n\} \cdot Pr\{n\}}{Pr\{S\}} = \frac{(v - \sqrt{2b})^{n-1} \cdot \exp(-v + \sqrt{2b})}{(n-1)!} \tag{2.10}$$

The term $n - 1$ in the final expression of Equation 2.10 reflects the departure of a successful packet from the system. The resulting distribution is Poisson with mean $v - \sqrt{2b}$.

The probability that a collision slot occurs is equal to the probability that two or more of the backlogged stations attempt transmission in the slot; that is;

$$\begin{aligned} Pr\{C \mid n\} &= \sum_{m=2}^{n} c(n,m) \cdot \left(\frac{\sqrt{2b}}{v}\right)^{m} \cdot \left(1 - \frac{\sqrt{2b}}{v}\right)^{n-m} \\ &= 1 - \sum_{m=0}^{1} c(n,m) \cdot \left(\frac{\sqrt{2b}}{v}\right)^{m} \cdot \left(1 - \frac{\sqrt{2b}}{v}\right)^{n-m} \\ &= 1 - \left(1 - \frac{\sqrt{2b}}{v}\right)^{n} - n \cdot \left(\frac{\sqrt{2b}}{v}\right) \cdot \left(1 - \frac{\sqrt{2b}}{v}\right)^{n-1} \end{aligned} \tag{2.11}$$

where the notation $c(n,m)$ is defined by

$$c(n,m) = \begin{cases} \dfrac{n!}{m! \cdot (n-m)!}; & n, m, (n-m) \geq 0 \\ \\ 0; & \text{otherwise} \end{cases} \tag{2.12}$$

Since only three outcomes are possible for each slot, the expected probability of a collision slot is

$$Pr\{C\} = 1 - Pr\{I\} - Pr\{S\} = 1 - (1 + \sqrt{2b}) \cdot \exp(-\sqrt{2b}) \tag{2.13}$$

Applying Bayes' Rule to Equations 2.1, 2.11 and 2.13 yields the probability distribution of n given a collision slot as

$$\begin{aligned} Pr\{n \mid C\} &= \frac{Pr\{C \mid n\} \cdot Pr\{n\}}{Pr\{C\}} \\ &= \frac{\exp(-v + \sqrt{2b})}{n! \cdot (\exp(\sqrt{2b}) - 1 - \sqrt{2b})} \\ &\quad \cdot [v^{n} - (v - \sqrt{2b})^{n} - n \cdot \sqrt{2b} \cdot (v - \sqrt{2b})^{n-1}] \end{aligned} \tag{2.14}$$

Although Equation 2.14 is not a Poisson distribution, $Pr\{n \mid C\}$ can be closely approximated by a Poisson distribution with mean $v + 2$, that is,

$$Pr\{n \mid C\} = \frac{(v+2)^n \cdot \exp(-v-2)}{n!} \tag{2.15}$$

Figure 2.2 plots the distributions given by Equations 2.14 and 2.15 for various values of v. The upper, labeled curves represent the actual distributions and the lower curves are the Poisson-approximated probability distributions. For all figures throughout the thesis this convention will be used. As seen from Figure 2.2, the Poisson approximation to the actual distribution is rather good and improves with increasing values of v. Therefore, when a collision occurs, the estimate of the expected number of backlogged stations is increased by 2.

After updating the estimate v of the number of backlogged stations, any new packets generated during the slot must be added to v to maintain accuracy in the estimate. On the average, the channel input rate λ provides the expected number of newly generated packets in the slot.

To summarize the pseudo-Bayesian algorithm used to update the estimate of the expected number of backlogged stations in the network, v will denote the estimate for slot k and v_{k+1} will denote the updated estimate used for the succeeding slot. The appropriate rule for updating the estimated backlog is [Ref. 1: p. 244]

$$v_{k+1} = \begin{cases} v_k \cdot (1 - q_r) + \lambda b; & \text{for idle} \\ v_k \cdot (1 - q_r) + \lambda(1 + b); & \text{for success} \\ v_k + 2 + \lambda(1 + b); & \text{for collision} \end{cases} \tag{2.16}$$

where q_r is given in Equation 2.4.

This rule is motivated by the fact that, if the a priori distribution of n is Poisson with mean v, then, given an idle slot, the a posteriori distribution of n_k is Poisson with mean $v_k \cdot (1 - q_r)$. Accounting for the Poisson arrivals in the idle slot

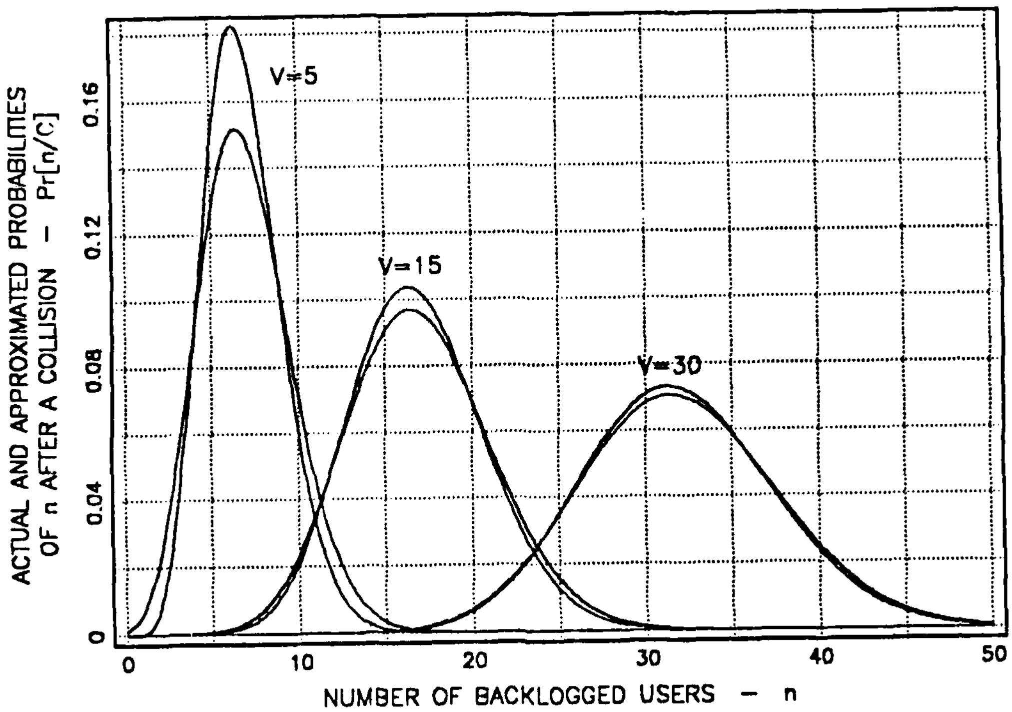

Figure 2.2: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users after a Collision Occurs in a Slotted CSMA Network

of duration b, the resulting distribution is Poisson with mean v_{k+1} as shown above. Similarly, given a successful transmission, the a posteriori distribution on $n-1$ (removing the successful packet) is Poisson with $v_k \cdot (1-q_r)$. Accounting for the Poisson arrivals in the successful slot and following idle slot, the final distribution is Poisson with mean v_{k+1} as shown. Finally, if a collision occurs, the a posteriori distribution of n_k is not quite Poisson, but is reasonably approximated as Poisson with mean $v+2$ as proved before. Adding $\lambda(1+b)$ for the new arrivals, the final expression is found.

When n and v are small, then q_r is large and new arrivals are scarcely delayed at all. When $n = v$ and n is large, the departure rate is approximately $1/(1+\sqrt{2b})$, so that for $\lambda < 1/(1+\sqrt{2b})$, the departure rate exceeds the arrival rate, and the

backlog decreases on the average. Finally, if $| n - v |$ is large, the expected change in backlog can be positive, but the expected change in $| n - v |$ is negative. Figure 2.3 provides a qualitative picture of the expected changes in n and $n - v$. Thus, if the system starts at some arbitrary point in the (n, v) plane, n might increase on the average for a number of slots, but eventually n and v will come closer together and then n will decrease on the average. In applications, the arrival rate λ is typically unknown and slowly varying. Thus, the algorithm must either estimate λ from the time average rate of successful transmissions or set its value within the algorithm to some fixed value $\lambda < 1/(1 + \sqrt{2b})$, then stability will be achieved.

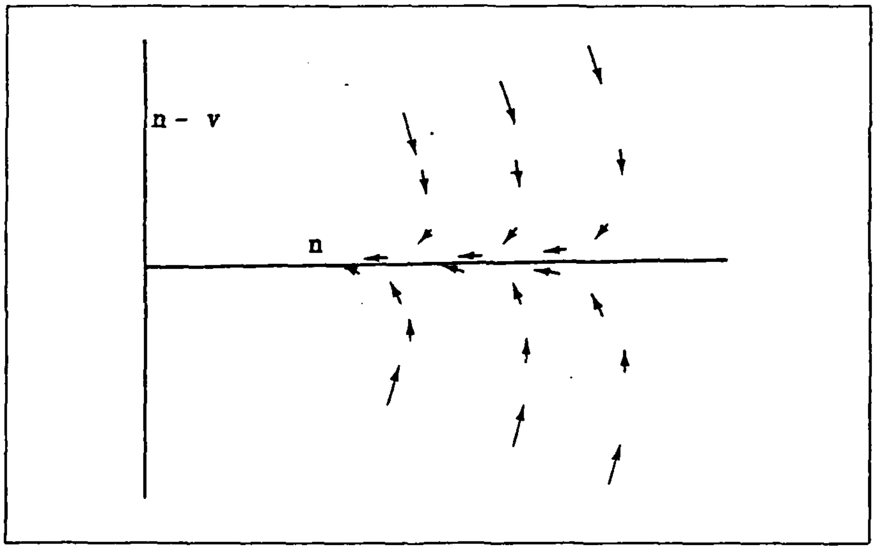

Figure 2.3: Drift of n and $n - v$ for the Pseudo-Bayesian Stabilization Algorithm. When the Absolute Value of $n - v$ is Large, it approaches Zero Faster than it Increases. [Ref. 1: p. 219]

B. PSEUDO-BAYESIAN STABILITY OF CSMA/CD

CSMA/CD can be analyzed in the same way as CSMA. It is assumed that each backlogged node transmits after each idle slot with the broadcast probability

q_r, and it is assumed at the outset that the number of nodes transmitting after an idle slot is Poisson with parameter

$$G = \lambda + q_r \cdot n \tag{2.17}$$

Considering state transitions at the ends of idle slots, if no transmissions occur, the next idle slot ends after time b. If one transmission occurs, the next idle slot ends after $1 + b$. To be able to correspond precisely to the model for idle slots, the packet durations should be multiples of the idle slot durations; thus the expected packet duration is 1. Finally, if a collision occurs, the next idle slot ends after $2b$; in other words, nodes must hear an idle slot after the collision to know that it is safe to transmit.

The throughput expression for CSMA/CD, that can be interpreted as the departure rate in state k is [Ref. 10: p. 248]

$$S = \frac{G \cdot \exp(-G)}{b + G \cdot \exp(-G) + b \cdot [1 - (1 + G) \cdot \exp(-G)]} \tag{2.18}$$

Figure 2.4 shows this ratio for $b = 0.01$. This expression is maximized over G at $G = 0.77$ and the resulting value of the right-hand side is $1/(1 + 3.31 \cdot b)$. Thus the broadcast probability q_r is chosen for a given estimated backlog v, as

$$q_r = \min\left\{\frac{0.77}{v}, 0.77\right\} \tag{2.19}$$

The probability that a successful transmission will occur is the same as in the CSMA case and is given by Equation 2.5. Using Equation 2.1 and averaging over the ensemble of possible values for the number of backlogged users if a successful transmission occurs, the probability of success is

$$Pr\{S\} = \sum_{n=1}^{\infty} Pr\{S \mid n\} \cdot Pr\{n\} = 0.77 \cdot \exp(-0.77) \tag{2.20}$$

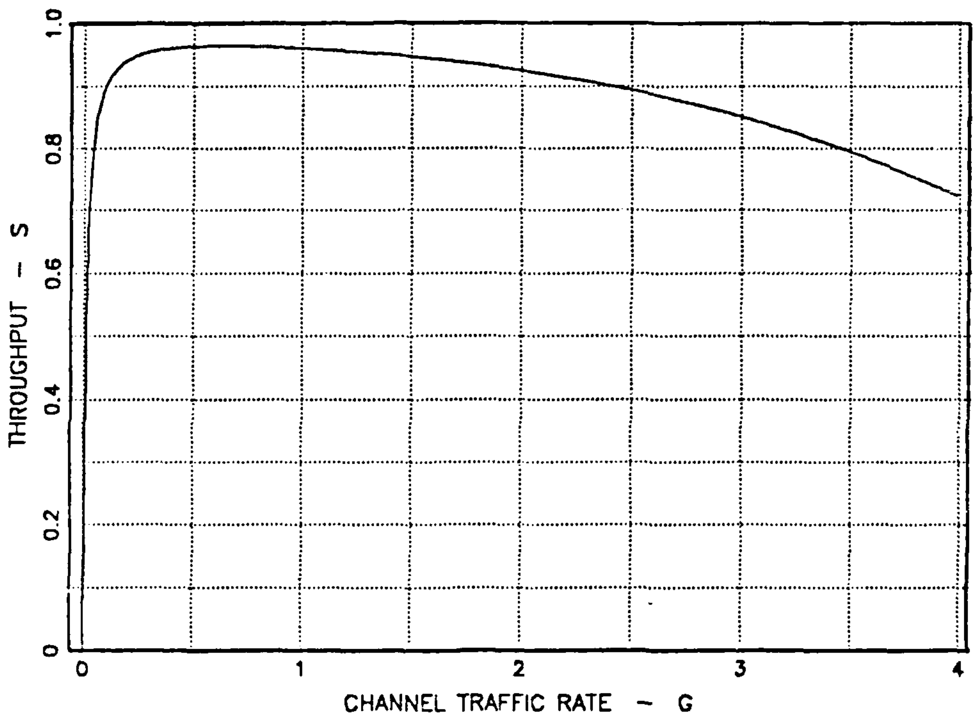

Figure 2.4: Throughput vs. Channel Traffic Rate in Slotted CSMA/CD. ($b = 0.01$)

The expected probability of an idle slot is determined by using Equation 2.1 and 2.7 from the previous section:

$$\begin{aligned} Pr\{I\} &= \sum_{n=0}^{\infty} Pr\{I \mid n\} \cdot Pr\{n\} \\ &= \sum_{n=0}^{\infty} \left(1 - \frac{0.77}{v}\right)^n \cdot \frac{v^n \cdot \exp(-v)}{n!} = \exp(-0.77) \end{aligned} \tag{2.21}$$

What remains is to describe the estimation algorithm for v. Using the a priori probability density function Equations 2.1, 2.5 and 2.20 for the backlogged stations, and applying Bayes' Rule, Equation 1.4, the a posteriori distribution of n given a successful transmission is

$$Pr\{n \mid S\} = \frac{Pr\{S \mid n\} \cdot Pr\{n\}}{Pr\{S\}} = \frac{(v - 0.77)^{n-1} \cdot \exp(-v + 0.77)}{(n-1)!} \tag{2.22}$$

The a posteriori distribution of n given an idle slot is

$$Pr\{n \mid I\} = \frac{Pr\{I \mid n\} \cdot Pr\{n\}}{Pr\{I\}} = \frac{(v - 0.77)^n \cdot \exp(-v + 0.77)}{n!} \tag{2.23}$$

The a posteriori distribution of n given a successful transmission or an idle slot is Poisson with mean

$$E\{n - 1 \mid S\} = E\{n \mid I\} = v_k \cdot (1 - q_r) \tag{2.24}$$

In the case of a collision, the expected probability of a collision slot is

$$Pr\{C\} = 1 - Pr\{I\} - Pr\{S\} = 1 - 1.77 \cdot \exp(-0.77) \tag{2.25}$$

and

$$\begin{aligned} Pr\{C \mid n\} &= \sum_{m=2}^{n} c(n,m) \cdot \left(\frac{0.77}{v}\right)^m \cdot \left(1 - \frac{0.77}{v}\right)^{n-m} \\ &= 1 - \sum_{m=0}^{1} c(n,m) \cdot \left(\frac{0.77}{v}\right)^m \cdot \left(1 - \frac{0.77}{v}\right)^{n-m} \\ &= 1 - \left(1 - \frac{0.77}{v}\right)^n - n \cdot \left(\frac{0.77}{v}\right) \cdot \left(1 - \frac{0.77}{v}\right)^{n-1} \end{aligned} \tag{2.26}$$

Thus the a posteriori distribution of n, given a collision slot is no longer Poisson, that is, using Equations 2.25 and 2.26

$$\begin{aligned} Pr\{n \mid C\} &= \frac{Pr\{C \mid n\} \cdot Pr\{n\}}{Pr\{C\}} \\ &= \frac{\exp(-v + 0.77)}{\exp(0.77) - 1.77} \left[v^n - (v - 0.77)^{n-1} \cdot (v - 0.77(1 - n))\right] \end{aligned} \tag{2.27}$$

but it can be approximated by a Poisson distribution with mean $v + 1.5$. Figure 2.5 shows that the actual and approximated distributions are similar and give better results for increasing values of v.

Were it not for collisions, the reproducing nature of the a priori probability density function would lead to an extremely simple, recursive Bayes estimate. In

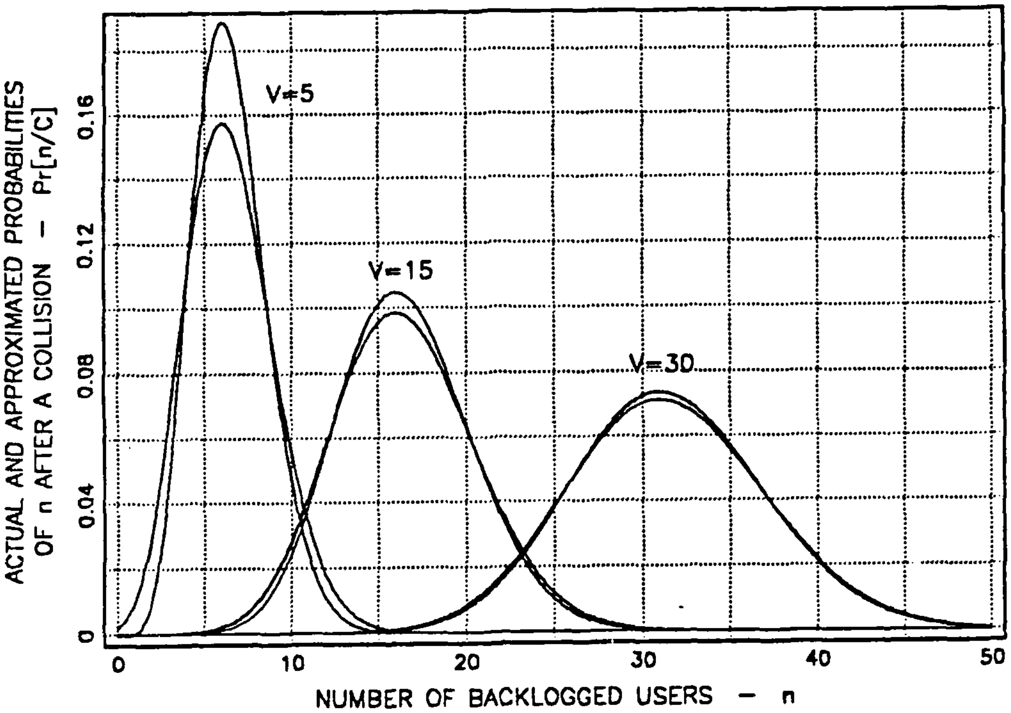

Figure 2.5: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a CSMA/CD Network.

spite of the non-Poisson nature of the a posteriori distribution after a collision, the algorithm for v_{k+1} proceeds as if the a posteriori distribution is Poisson. To this approximation of the a posteriori probability density function the term pseudo-Bayesian is attached.

Accounting for the expected number of new arrivals leads to the following for v_{k+1}

$$v_{k+1} = \begin{cases} v_k \cdot (1 - q_r) + \lambda b; & \text{for idle} \\ v_k \cdot (1 - q_r) + \lambda(1 + b); & \text{for success} \\ v_k + 1.5 + \lambda(1 + b); & \text{for collision} \end{cases} \tag{2.28}$$

where q_r is given in Equation 2.19. The pseudo-Bayesian slotted CSMA/CD is very similar in behavior to pseudo-Bayesian slotted CSMA. The values of n and v char-

acterize the system. Each of the backlogged packets is independently transmitted with the packet broadcast probability q_r, if the number of backlogs is large and $n = v$. Therefore, the channel traffic rate G is one packet per slot and throughput is maximized. For the values of $n > v$, the channel traffic rate will be larger than one packet per slot. Thus, collisions will occur more frequently than idle or success slots. In this case v grows at a faster rate than n. The difference $n - v$ converges to zero and throughput approaches the maximum achievable. Idle and success slots will occur more frequently in the case of $n < v$, where the channel traffic rate will be smaller than one packet per slot. The reduction in the estimate due to idle slots will cause v to decrease more rapidly than n. The difference $n - v$ again converges to zero and the throughput increases towards the maximum. Figure 2.3 again provides a qualitative picture of the expected changes in n and $n - v$.

C. STABILIZATION OF CSMA AND CSMA/CD WITH TWO POWER LEVELS

In local area networks and short range terrestrial radio networks where power is not a constraint, the use of random power levels has been shown to improve the throughput. The reason for the improvement is an inherent effect called power capture which is the ability of a receiver to receive correctly the strongest of several messages that arrive during overlapping intervals. Data packets from all stations are assumed to arrive at the receiver after spatial attenuation with one of the two power levels contained in the set $\Omega = \{1, M\}$. Each user randomly chooses one of the equally likely transmitted power levels from the set Ω for each packet transmitted or retransmitted. Therefore, the equal opportunity for each station to successfully communicate with the receiver is preserved, and priority classes among the users are avoided. [Ref. 5: p. 1026]

To determine the optimal broadcast probability q_r in slotted CSMA networks using two transmitted signal power levels $\Omega = \{1, M\}$, the expected probability of a success slot is considered, where the broadcast probability q_r is set equal to G_s/v. G_s is the channel traffic rate which maximizes the throughput. The probability of success slot with n backlogged stations in the network transmitting independently with the broadcast probability q_r is given by

$$Pr\{S \mid n\} = \sum_{m=1}^{n} c(n,m) \cdot q_r^m \cdot (1-q_r)^{n-m} \cdot Pr\{\text{capture} \mid m\} \tag{2.29}$$

If only one packet exists in the slot, it will be successfully received and captured by the receiver. In the case of more than one data packet, the receiver may capture of the transmitted packets which has the higher power level M. All the interfering packets have the lower power level 1. The maximum number of interfering packets can be limited by the number $N + \lfloor M/\gamma_0 \rfloor + 1$ where γ_0 is the received threshold level and $\lfloor x \rfloor$ denotes the integer part of x. Therefore, given m interfering packets, the capture probability of one packet is, assuming random selection of power levels,

$$Pr\{\text{capture} \mid m\} = \begin{cases} 1; & m = 1 \\ \frac{m}{2^m}; & 2 \le m \le N+1 \\ 0; & m > N+1 \end{cases} \tag{2.30}$$

Therefore,

$$Pr\{S \mid n\} = n \cdot \left(\frac{G_s}{v}\right) \cdot \left(1 - \frac{G_s}{v}\right)^{n-1} + \sum_{m=2}^{N+1} c(n,m) \cdot \left(\frac{G_s}{v}\right)^m \cdot \left(1 - \frac{G_s}{v}\right)^{n-m} \cdot \frac{m}{2^m} \tag{2.31}$$

To get the expression for the average probability of a success slot, Equation 2.31 will be averaged over all possible values of the number of backlogged stations; that is,

$$Pr\{S\} = \sum_{n=1}^{\infty} Pr\{S \mid n\} \cdot Pr\{n\}$$

$$
\begin{aligned}
&= \sum_{n=1}^{\infty} \frac{v^n \cdot \exp(-v)}{n!} \cdot \left[n \cdot \left(\frac{G_s}{v}\right) \cdot \left(1 - \frac{G_s}{v}\right)^{n+1} + \sum_{m=2}^{N+1} c(n,m) \cdot \left(\frac{G_s}{v}\right)^m \right. \\
&\qquad \left. \cdot \left(1 - \frac{G_s}{v}\right)^{n-m} \cdot \frac{m}{2^m} \right] \\
&= G_s \cdot \exp(-G_s) \cdot \left[1 + \frac{1}{2} \sum_{m=1}^{N} \frac{(G_s/2)^m}{m!} \right] \qquad (2.32)
\end{aligned}
$$

Applications of Bayes' Rule to Equations 2.1, 2.31 and 2.32 yields the probability distribution of n conditioned on a success slot given by

$$
\begin{aligned}
Pr\{n \mid S\} &= \frac{Pr\{S \mid n\} \cdot Pr\{n\}}{Pr\{S\}} \\
&= \frac{(v - G_s)^{n-1} \cdot \exp(-v + G_s)}{(n-1)!} \\
&\quad \cdot \frac{\left[1 + \frac{1}{2n} \sum_{m=2}^{N+1} c(n,m) \left(\frac{G_s}{2}\right)^{m-1} \cdot (v - G_s)^{1-m} \cdot m \right]}{\left[1 + \frac{1}{2} \sum_{m=1}^{N} \frac{(G_s/2)^m}{m!} \right]} \qquad (2.33)
\end{aligned}
$$

To preserve the Poisson distribution of the number of backlogged packets, $Pr\{n \mid S\}$ given by Equation 2.33 can be approximated by

$$
Pr\{n \mid S\} = \frac{(v - \sqrt{2b})^{n-1} \cdot \exp(-v + \sqrt{2b})}{(n-1)!} \qquad (2.34)
$$

which is the case in single power level CSMA.

When n backlogged stations exist in the network and independently transmit their data packets in each slot with probability $q_r = G_s/v$, the probability that an idle slot will occur is given by

$$
Pr\{I \mid n\} = \left(1 - \frac{G_s}{v}\right)^n \qquad (2.35)
$$

Averaging over the number of backlogged stations yields the expected probability of an idle slot; that is,

$$
\begin{aligned}
Pr\{I\} &= \sum_{n=0}^{\infty} Pr\{I \mid n\} \cdot Pr\{n\} \\
&= \sum_{n=0}^{\infty} \left(1 - \frac{G_s}{v}\right)^n \cdot \frac{v^n \cdot \exp(-v)}{n!} \\
&= \exp(-G_s) \qquad (2.36)
\end{aligned}
$$

Applying Bayes' Rule to Equations 2.1, 2.35 and 2.36, gives the probability distribution of the number of backlogged users after an idle slot as

$$\begin{aligned} Pr\{n \mid I\} &= \frac{Pr\{I \mid n\} \cdot Pr\{n\}}{Pr\{I\}} \\ &= \frac{(v - G_s)^n \cdot \exp(-v + G_s)}{n!} \end{aligned} \tag{2.37}$$

The a posteriori distribution of n given an idle slot is Poisson with mean $\max\{v - G_s, 0\}$. In the case of an idle slot, users reduce their estimate of the expected number of backlogged stations by G_s, if v is already less than G_s, the mean will be set to zero.

The expected probability of a collision slot is

$$\begin{aligned} Pr\{C\} &= 1 - Pr\{I\} - Pr\{S\} \\ &= 1 - \exp(-G_s) - G_s \cdot \exp(-G_s) \cdot \left[1 + \frac{1}{2}\sum_{m=1}^{N} \frac{(G_s/2)^m}{m!}\right] \end{aligned} \tag{2.38}$$

Because of the complexity of Equation 2.38 for the distribution of the backlogged stations after a collision slot, the total probability theorem is used. This is

$$Pr\{n\} = Pr\{n \mid I\} \cdot Pr\{I\} + Pr\{n \mid S\} \cdot Pr\{S\} + Pr\{n \mid C\} \cdot Pr\{C\} \tag{2.39}$$

Thus, the distribution of the backlogged users after a collision slot is

$$\begin{aligned} Pr\{n \mid C\} &= \frac{Pr\{n\} - Pr\{n \mid I\} \cdot Pr\{I\} - Pr\{n \mid S\} \cdot Pr\{S\}}{Pr\{C\}} \\ &= \frac{\dfrac{\exp(-v)}{n!} \cdot \left[v^n - (v - G_s)^{n-1} \cdot \left[v + (n-1) \cdot G_s + \sum_{m=2}^{N+1} c(n,m) \cdot \left(\dfrac{G_s}{2}\right)^m \cdot (v - G_s)^{1-m} \cdot m \right] \right]}{1 - \exp(-G_s) \cdot \left[1 + G_s \left[1 + \dfrac{1}{2}\sum_{m=2}^{N+1} \dfrac{(G_s/2)^{m-1}}{(m-1)!}\right]\right]} \end{aligned} \tag{2.40}$$

This probability distribution can be approximated by a Poisson distribution with mean $v + 2$ such as in a single power level, that is

$$Pr\{n \mid C\} = \frac{(v+2)^n \cdot \exp(-v-2)}{n!} \quad (2.41)$$

In Figure 2.6 the actual and Poisson approximating probability distributions are shown for slotted CSMA with two power levels ($G_s = 0.2$) which are pretty similar. Thus, the estimate of the expected number of backlogged users will be incremented by two when feedback indicates a collision slot occurred.

The updated estimated v_{k+1} for slotted CSMA will be

$$v_{k+1} = \begin{cases} v_k \cdot (1 - q_r) + \lambda b; & \text{for idle} \\ v_k \cdot (1 - q_r) + \lambda(1 + b); & \text{for success} \\ v_k + 2 + \lambda(1 + b); & \text{for collision} \end{cases} \quad (2.42)$$

where q_r is given by

$$q_r = \min\left\{\frac{G_s}{v}, 1\right\} \quad (2.43)$$

The pseudo-Bayesian algorithm, as adapted to slotted CSMA systems using two power levels, should perform in a similar manner to the single power level slotted CSMA case. The system should stabilize at the appropriate maximum achievable throughput $S_{\max}$, which is maximized for $G_s = 0.2$ value.

Following the same procedure for the slotted CSMA/CD case, identical distributions are found. The throughput of slotted CSMA/CD with two power levels is maximized for a G_s value, which is this time greater than one. [Ref. 3: p.18] Thus the distribution of the backlogged users after a collision slot is like that in Equation 2.40 for the $G_s = 2$ value of slotted CSMA/CD with two power level, but

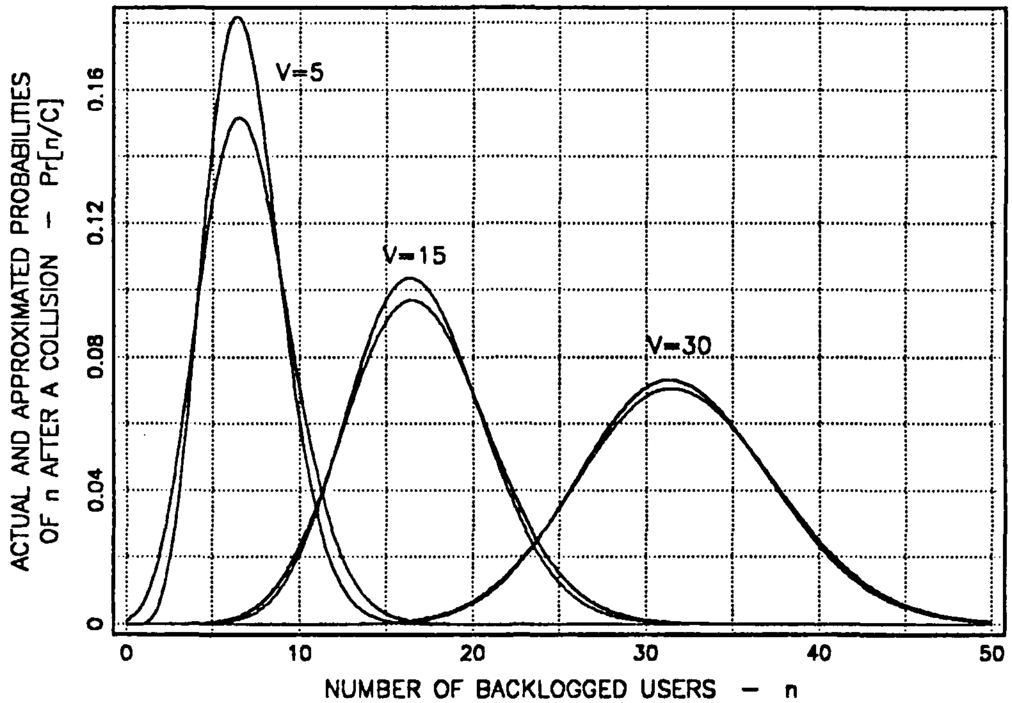

Figure 2.6: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a Slotted CSMA Network with Two Power Levels. $(G_s = 0.2)$

approximated by a Poisson process with mean $v + 1.5$, such as in the single power levels slotted CSMA/CD; that is

$$Pr\{n \mid C\} = \frac{(v+1.5)^n \cdot \exp(-v-1.5)}{n!} \tag{2.44}$$

Figure 2.7 shows both distributions for $G_s = 2$.

The updated estimated for the slotted CSMA/CD with two power levels is

$$v_{k+1} = \begin{cases} v_k \cdot (1-q_r) + \lambda b; & \text{for idle} \\ v_k \cdot (1-q_r) + \lambda(1+b); & \text{for success} \\ v_k + 1.5 + \lambda(\delta + b); & \text{for collision} \end{cases} \tag{2.45}$$

for which q_r is

$$q_r = \min\left\{\frac{G_s}{v}, 1\right\} \tag{2.46}$$

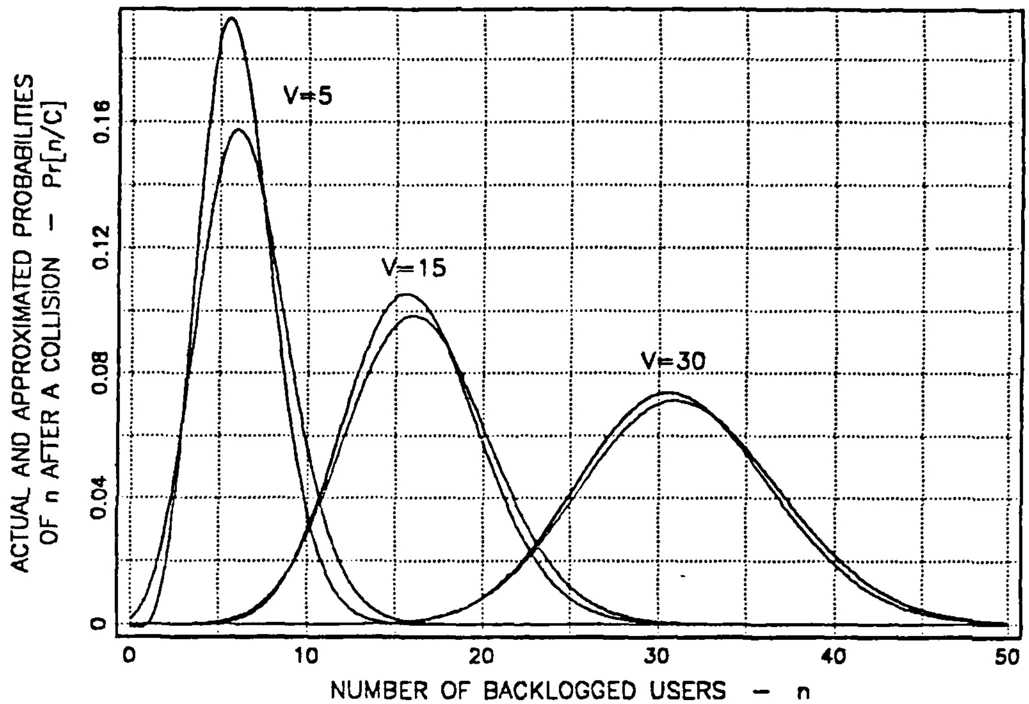

Figure 2.7: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users after a Collision Occurs in a Slotted CSMA/CD Network with Two Power Levels. $(G_s = 2)$

It is shown that for both slotted CSMA and CSMA/CD pseudo-Bayesian algorithm performed in a similar manner such as in one power level. The stabilization at the maximum throughput $S_{\max}$ can be accomplished for slotted CSMA with two power levels at channel traffic rates G less than one, and for slotted CSMA/CD with two power levels at channel traffic rate G greater than one.

Whether stabilizing one or two power level CSMA and CSMA/CD networks, the pseudo-Bayesian algorithm allows growth of the user population without any major complications. Since all users maintain the same estimate of the expected number of backlogged users, inclusion of this information in the required overhead information for each transmitted packet would allow users new to the network to

synchronize quickly. Alternatively, the receiver could provide its computed value of the estimate in the feedback for success slots. [Ref. 5: p. 8]

III. PSEUDO-BAYESIAN STABILITY FOR SLOTTED MULTICHANNEL BROADCAST NETWORKS

A. MULTICHANNEL CSMA AND CSMA/CD PROTOCOLS

The use of a multiple access broadcast channel implies that network users must follow some rules that discipline bus accesses. The set of rules to be followed constitutes the channel access scheme, whose goal is to allow an efficient utilization of the communication channel such as CSMA and CSMA/CD. The use of CSMA and CSMA/CD protocols in local area networks, coupled with the high number of bus insertions, limits the data rate that can be used on a given distance, and thus limits the network capacity. To achieve a high capacity using a reliable network architecture, multichannel local area networks (M–LAN's) are considered. M–LAN's use a set of lower data rate parallel buses connected all stations. Buses need not be physically separate; they can be obtained by dividing the bandwidth of a single physical connection. M–LAN's offer the reliability characteristic typical of bus structured local area networks, while providing a large bandwidth than can be used for data packet transmission and employing moderate speed parallel channels. The M–LAN architecture is completely modular, so that it allows a gradual system growth following user needs.

A M–LAN is made of a set of M parallel broadcast channels to which N stations are connected, each one by means of M separate interfaces, one for each channel. Broadcast channels are assumed to be of same bandwidth; W_i being the

bandwidth of the ith channel in Hz. Data packets are assumed to be of constant length, thus the packet transmission time on the ith channel T_i is inversely proportional to W_i. All channels have the same bandwidth $W_i = W/M$, $i = 1, 2, \cdots, M$, and if T_0 is the time needed to transmit a packet on a single broadcast channel with the total available bandwidth W, then

$$T_i = M \cdot T_0 \tag{3.1}$$

for all channels. The channel end-to-end propagation and detection delay is assumed to be A s. The normalized propagation delays

$$b = \frac{A}{T_0} \tag{3.2}$$

and

$$b_i = \frac{A}{T_i} \tag{3.3}$$

will be used throughout the calculations. The total data traffic offered to the broadcast system, including new and rescheduled packets is assumed to be Poisson with rate v packets/s. Measuring time in T_0 units the offered traffic is

$$g = v \cdot T_0 \tag{3.4}$$

The stochastic properties of the traffic offered to each channel depend on the policy followed by each station in the selection of a channel on which the packet transmission is scheduled. If channels are randomly chosen, independently of their state, then the offered traffic to the ith channel is Poisson with rate $v \cdot P_i$, where P_i is the probability of choosing channel i. If a channel is chosen only among those that are sensed idle, then the offered traffic on each channel is not Poisson. The evaluation of throughput S_i of channel i is greatly simplified if it is assumed that

the traffic offered to channel i is Poisson. In this case S_i is evaluated independently of any other channel, using the single channel expression with offered traffic

$$G_i = v \cdot P_i \cdot T_i = g \cdot P_i \cdot T_i/T_0 = g \cdot P_i \cdot W/W_i \tag{3.5}$$

It is assumed that stations are able to detect the presence of a carrier on each one of the M channels simultaneously. This allows the use of CSMA type protocols. Moreover it is assumed that each interference is able to detect a collision on the channel on which it is transmitting, so that CSMA/CD type protocols can be used for transmission of data packets in M–LAN's. Each station is assumed to be able to transmit on one channel and simultaneously receive from several other channels. No thermal noise is considered so that packets are not correctly received only when they collide. No capture effects are assumed for different received signal powers.

The extension to multiple channel systems implies that a station must choose a channel where the packet is transmitted. Several different ways of choosing the channel can be envisioned; two of them will be considered.

1. A station with a ready packet randomly chooses a channel before sensing it; this procedure is named random choice (RC) and thus the resulting protocols are named M–CSMA–RC and M–CSMA/CD–RC.

2. A station with a ready packet randomly chooses a channel among those that are sensed idle; this procedure is named idle channel choice (IC) and originates the M–CSMA–IC and M–CSMA/CD–IC protocols.

B. M–CSMA–RC AND M–CSMA/CD–RC

Channel i is randomly chosen with probability

$$P_i = \frac{1}{M} \tag{3.6}$$

thus the offered traffic to the ith channel is Poisson with rate G_i packets per T_i second as in Equation 3.5. The total system throughput is evaluated using

$$S = g \cdot P_s \tag{3.7}$$

where P_s is the probability that a packet is successfully transmitted and given by

$$P_s = \sum_{i=1}^{M} P_{si} \cdot P_i \tag{3.8}$$

where P_{si} is the success probability on the ith channel, conditioned by the choice of channel i, and depends on b_i, the propagation on channel i normalized to the transmission time on the same channel. [Ref. 7: p. 376]

$$b_i = b \cdot W_i/W \tag{3.9}$$

For the slotted CSMA [Ref. 2: p. 314]

$$P_{si} = \frac{b_i \cdot \exp(-b_i \cdot G_i)}{b_i + 1 - \exp(-b_i \cdot G_i)} \tag{3.10}$$

which is the probability of success in the case of a single channel with offered traffic G_i and propagation delay b_i. Thus the total system throughput is obtained from Equation 3.7 by substituting Equation 3.8 for the success probability, Equation 3.10 for the success probability on the ith channel and Equation 3.5 for the offered traffic on the ith channel

$$S = g \cdot \sum_{i=1}^{M} \frac{b_i \cdot P_i \cdot \exp(-g \cdot b_i \cdot W \cdot P_i/W_i)}{b_i + 1 - \exp(-g \cdot b_i \cdot W \cdot P_i/W_i)} \tag{3.11}$$

Due to M equal channels assumption $W_i = W/M$, $b_i = b/M$, and $P - i = 1/M$, the total throughput for slotted M-CSMA is

$$S = \frac{g \cdot b \cdot \exp(-g \cdot b/M)}{M\,(b/M + 1 - \exp(-g \cdot b/M))} \tag{3.12}$$

which is the same relation that would be obtained with a single channel of bandwidth W and propagation delay b/M. For convenience $g \cdot b$ is denoted by G and with some simple manipulations Equation 3.12 becomes

$$S = \frac{G \cdot \exp(-G/M)}{b + M\,(1 - \exp(-G/M))} \tag{3.13}$$

Figure 3.1 shows the throughput of slotted M-CSMA protocol for M values 1, 2, 3, 5 and 10. ($b = 0.01$)

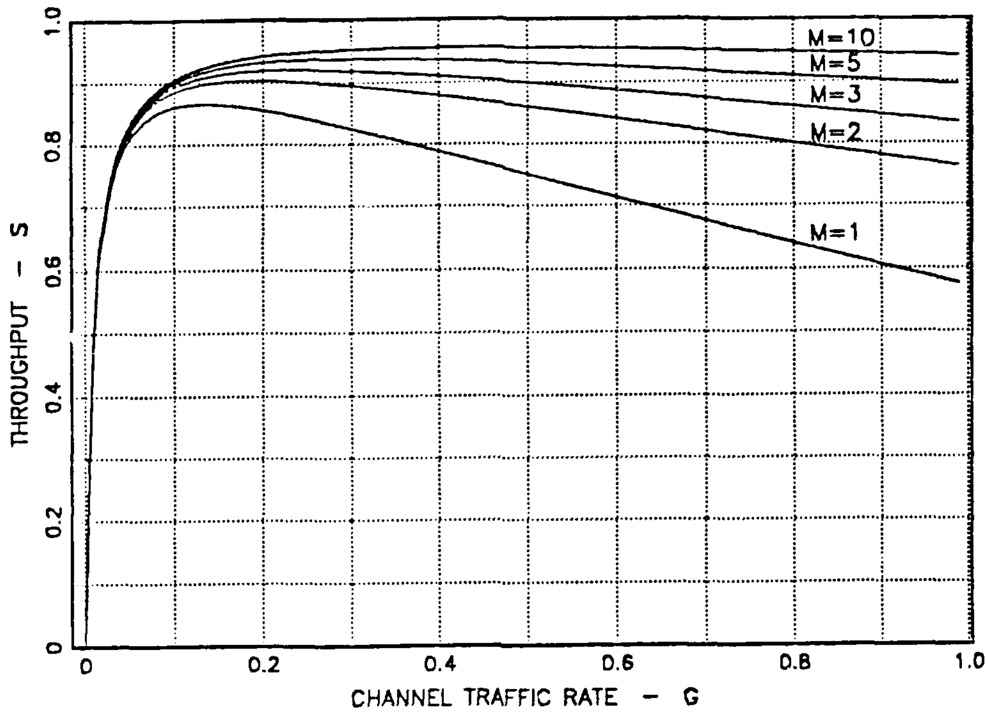

Figure 3.1: Throughput vs. Channel Traffic Rate in Slotted M–CSMA Networks.

For slotted M–CSMA/CD the same procedure is followed, and the total system throughput is obtained. [Ref. 7: p. 376]

$$S = \frac{G \cdot \exp(-G/M)}{b + M \cdot \delta[1 - \exp(-G/M) - (G/M) \cdot \exp(-G/M)] + G \cdot \exp(-G/M)} \qquad (3.14)$$

δ being an integer multiple of idle period length b. Figure 3.2 plots the throughput of slotted M-CSMA/CD for the same M values 1, 2, 3, 5, and 10. ($b = 0.01; \delta = 0.02$) The splitting of a single slotted CSMA or CSMA/CD channel into M parallel links according to M-CSMA-RC or M-CSMA/CD-RC yields a throughput increase due to the reduction of the normalized propagation delay on each channel.

To be able to proceed with pseudo-Bayesian stabilization, the value G, which maximizes the throughput, is obtained. For the slotted M-CSMA-RC protocol

$$G_{\max} = M\sqrt{\frac{2b}{M+b}} \qquad (3.15)$$

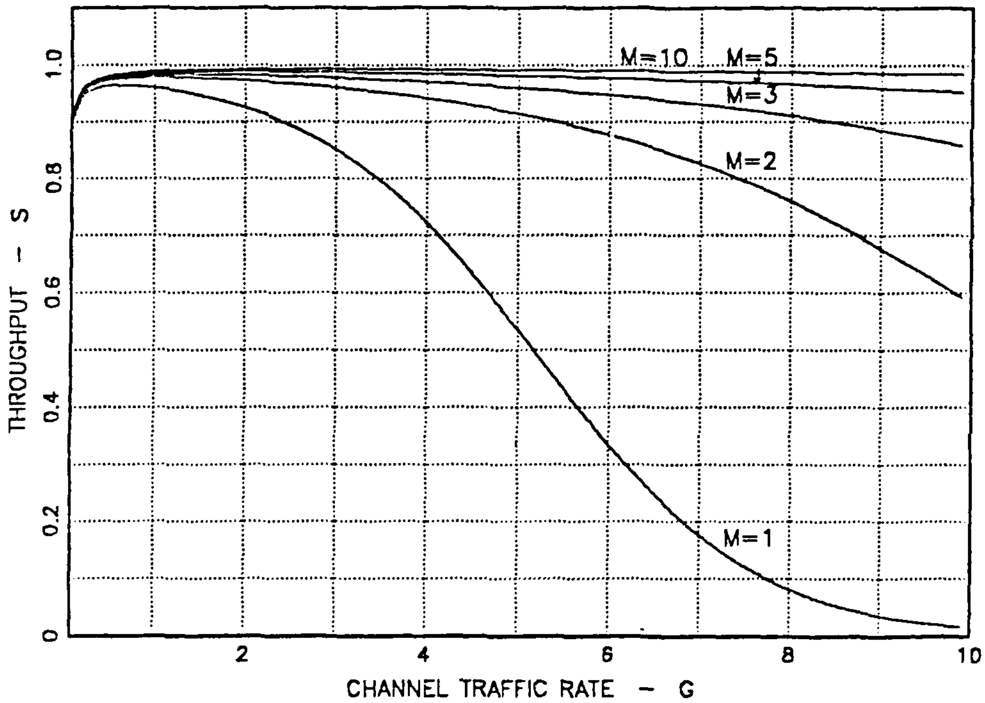

Figure 3.2: Throughput vs. Channel Traffic Rate in Slotted M–CSMA/CD Networks.

and for the slotted M–CSMA/CD–RC case

$$G_{\max} = M\sqrt{\frac{2b}{M\delta + b}} \tag{3.16}$$

the optimal broadcast probability on ith channel for slotted M–CSMA–RC networks is

$$q_i = \min\{\frac{M}{v}\sqrt{\frac{2b}{M+b}}, 1\} \tag{3.17}$$

For n packets in the system, with each packet independently transmitted in a slot with probability q_i, the probability of an idle slot,

$$Pr\{I \mid n\} = (1 - q_i)^n = \left(1 - \frac{M}{v}\sqrt{\frac{2b}{M+b}}\right) \tag{3.18}$$

The expected probability of an idle slot on the ith channel is determined by averaging over n; that is,

$$
\begin{aligned}
Pr\{I\} &= \sum_{n=0}^{\infty} Pr\{I \mid n\} \cdot Pr\{n\} \\
&= \exp\left(-M\sqrt{\frac{2b}{M+b}}\right) \qquad (3.19)
\end{aligned}
$$

Using the results above and Equation 2.1

$$
\begin{aligned}
Pr\{n \mid I\} &= \frac{Pr\{I \mid n\} \cdot Pr\{n\}}{Pr\{I\}} \\
&= \frac{\left(v - M\sqrt{\frac{2b}{M+b}}\right)^{n} \cdot \exp\left(-v + M\sqrt{\frac{2b}{M+b}}\right)}{n!} \qquad (3.20)
\end{aligned}
$$

The probability that a successful transmission will occur on the ith channel is

$$
Pr\{S \mid n\} = n \cdot q_i \cdot (1 - q_i)^{n-1} \qquad (3.21)
$$

which gives, with Equation 3.1, the following success probability.

$$
\begin{aligned}
Pr\{S\} &= \sum_{n=1}^{\infty} Pr\{S \mid n\} \cdot Pr\{n\} \\
&= M \cdot \sqrt{\frac{2b}{M+b}} \cdot \exp\left(-M\sqrt{\frac{2b}{M+b}}\right) \qquad (3.22)
\end{aligned}
$$

Thus, using Bayes' Rule

$$
\begin{aligned}
Pr\{n \mid S\} &= \frac{Pr\{S \mid n\} \cdot Pr\{n\}}{Pr\{S\}} \\
&= \frac{\left(v - M\sqrt{\frac{2b}{M+b}}\right)^{n-1} \cdot \exp\left(-v + M\sqrt{\frac{2b}{M+b}}\right)}{(n-1)!} \qquad (3.23)
\end{aligned}
$$

Since only three outcomes are possible for each slot, the expected probability of a collision slot is

$$
\begin{aligned}
Pr\{C\} &= 1 - Pr\{I\} - Pr\{S\} \\
&= 1 - \exp\left(-M\sqrt{\frac{2b}{M+b}}\right) - M\sqrt{\frac{2b}{M+b}} \cdot \exp\left(-M\sqrt{\frac{2b}{M+b}}\right) \qquad (3.24)
\end{aligned}
$$

The probability that more than one of the backlogged stations attempt transmission (i.e., the probability of a collision slot) is

$$
\begin{aligned}
Pr\{C \mid n\} &= \sum_{m=2}^{n} c(n,m) \cdot q_i^m \cdot (1-q_i)^{n-m} \\
&= 1 - \sum_{m=0}^{1} c(n,m) \cdot q_i^m \cdot (1-q_i)^{n-m} \\
&= 1 - \left(1 - \frac{M}{v}\sqrt{\frac{2b}{M+b}}\right)^n - n \cdot \left(\frac{M}{v}\sqrt{\frac{2b}{M+b}}\right) \\
&\quad \cdot \left(1 - \frac{M}{v}\sqrt{\frac{2b}{M+b}}\right)^{n-1}
\end{aligned} \tag{3.25}
$$

Applying Bayes' Rule to Equation 2.1, 3.24 and 3.25

$$
\begin{aligned}
Pr\{n \mid C\} &= \frac{Pr\{C \mid n\} \cdot Pr\{n\}}{Pr\{C\}} \\
&= \frac{\exp\left(-v + M\sqrt{\frac{2b}{M+b}}\right)}{n! \cdot \left[\exp\left(M\sqrt{\frac{2b}{M+b}}\right) - 1 - M\sqrt{\frac{2b}{M+b}}\right]} \\
&\quad \cdot \left[\begin{array}{l} v^n - \left(v - M\sqrt{\frac{2b}{M+b}}\right)^n \\ \qquad -n \cdot M\sqrt{\frac{2b}{M+b}} \cdot \left(v - M\sqrt{\frac{2b}{M+b}}\right)^{n-1} \end{array}\right]
\end{aligned} \tag{3.26}
$$

which is not a Poisson distribution, but can be closely approximated by a Poisson distribution for $M = 1$ with mean $(v + 2\sqrt{M})$,

$$
Pr\{n \mid C\} = \frac{(v + 2\sqrt{M})^n \cdot \exp(-v - 2\sqrt{M})}{n!} \tag{3.27}
$$

and for the values $M > 1$ with mean $v + 2(\sqrt{M} - 0.25M)$.

$$
Pr\{n \mid C\} = \frac{\left(v + 2(\sqrt{M} - 0.25M)\right) \cdot \exp\left(-v - 2(\sqrt{M} - 0.25M)\right)}{n!} \tag{3.28}
$$

Figure 3.3 shows the actual and Poisson approximating distributions for $M = 1$. Figure 3.4 for $M = 5$ and Figure 3.5 for $M = 10$. As seen from Figures 3.3, 3.4 and 3.5 the Poisson approximations to the actual distribution is rather good and improves with increasing values of v.

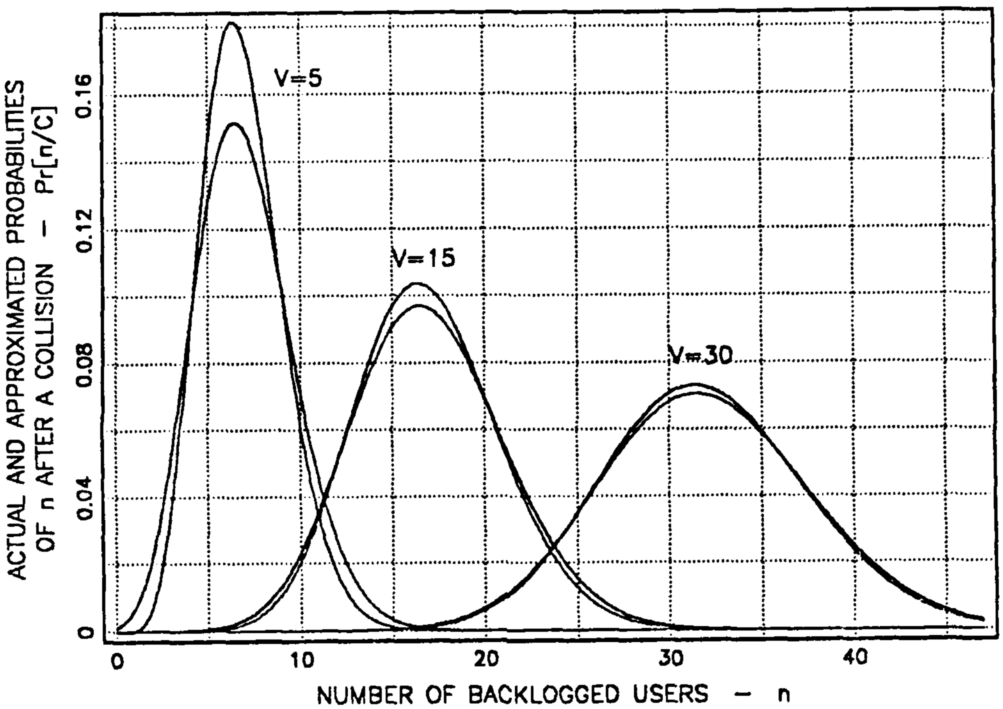

Figure 3.3: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a M–CSMA-RC Network for $M = 1$.

Updating the estimate v of the number of backlogged stations according to the pseudo-Bayesian method outlined above, any new packets generated during the slot will be added to v, the obtained updated estimate is

$$v_{k+1} = \begin{cases} v_k \cdot (1 - q_i) + \lambda b; & \text{for idle} \\ v_k \cdot (1 - q_i) + \lambda(1 + b); & \text{for success} \\ v_k + 2\sqrt{M} + \lambda(1 + b); & \text{for collision } (M = 1) \\ v_k + 2(\sqrt{M} - 0.25M) + \lambda(1 + b); & \text{for collision } (M > 1) \end{cases} \quad (3.29)$$

After determining the estimate v_{k+1}, users transmit packets ready for transmission in slot $k + 1$ with probability $1/v_{k+1}$.

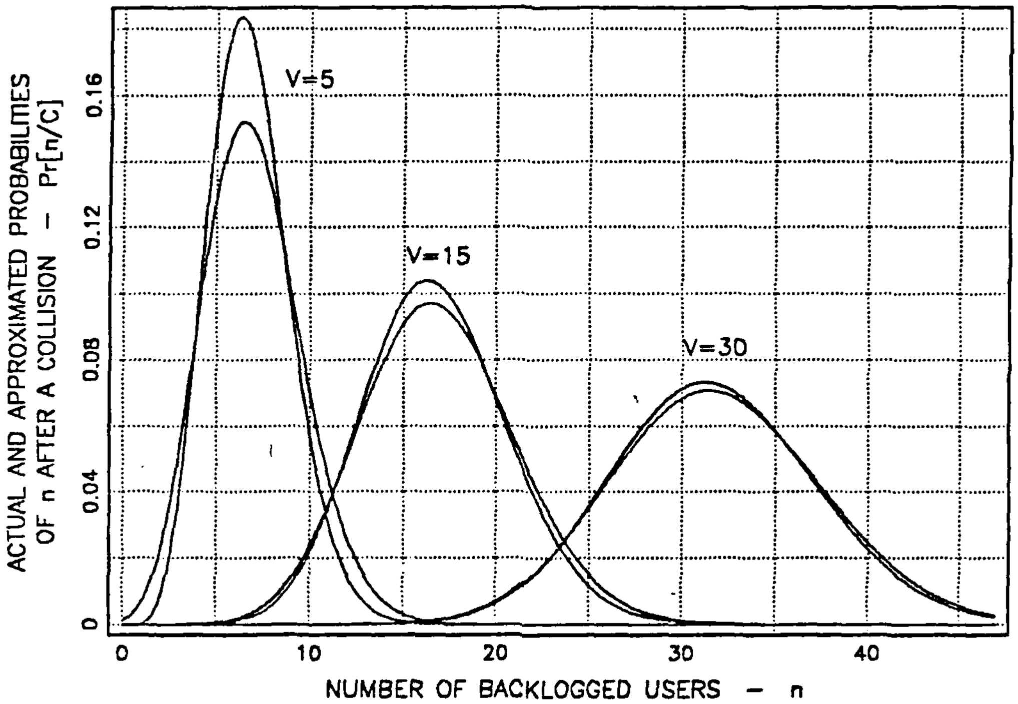

Figure 3.4: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a M–CSMA–RC Network for $M = 5$.

For the slotted M-CSMA/CD-RC the broadcast probability using Equation 3.16 is obtained as

$$q_i = \min\{\frac{M}{v}\sqrt{\frac{2b}{M\delta + b}}, 1\} \tag{3.30}$$

The probability distributions of n after a success or an idle slot are obtained as in M-CSMA-RC case, and are

$$Pr\{n \mid I\} = \frac{\left(v - M\sqrt{\frac{2b}{M\delta+b}}\right) \cdot \exp\left(-v + M\sqrt{\frac{2b}{M\delta+b}}\right)}{(n-1)!} \tag{3.31}$$

$$Pr\{n \mid S\} = \frac{\left(v - M\sqrt{\frac{2b}{M\delta+b}}\right)^{n-1} \cdot \exp\left(-v + M\sqrt{\frac{2b}{M\delta+b}}\right)}{(n-1)!} \tag{3.32}$$

As seen, the only difference between the given probability distributions is the contribution of the collision detection interval δ in M-CSMA/CD-RC, which equals one in the M-CSMA-RC case. The probability distribution of n given a collision

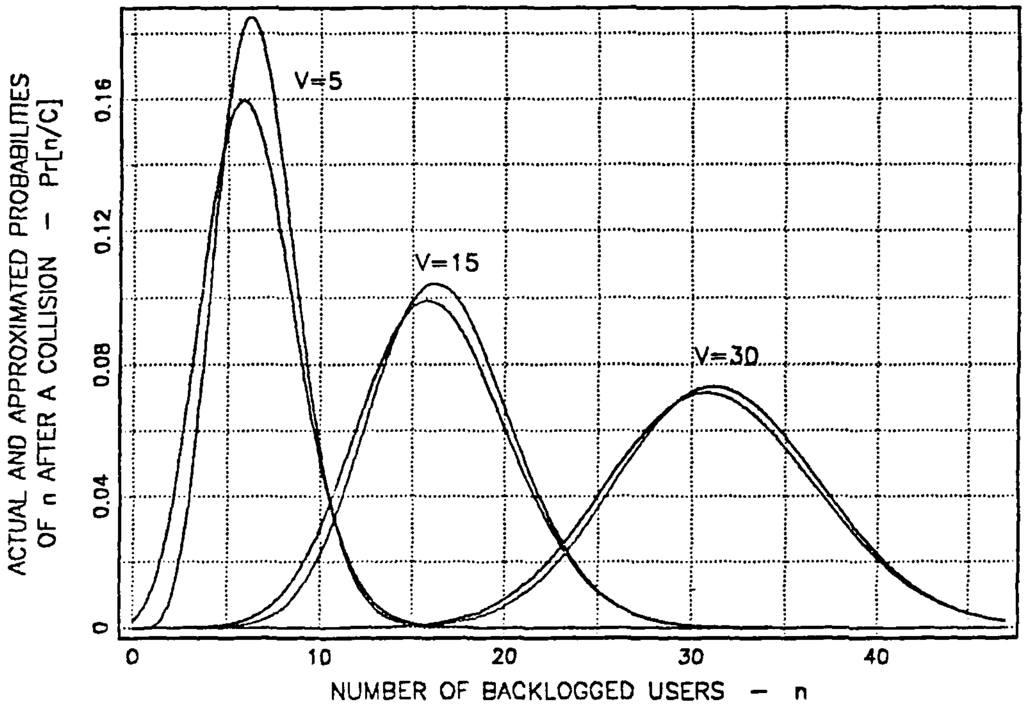

Figure 3.5: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a M–CSMA–RC Network for $M = 10$.

slot for M-CSMA/CD-RC becomes

$$Pr\{n \mid C\} = \frac{\exp\left(-v + M\sqrt{\frac{2b}{M\delta+b}}\right)}{n! \cdot \left[\exp\left(M\sqrt{\frac{2b}{M\delta+b}}\right) - 1 - M\sqrt{\frac{2b}{M\delta+b}}\right]} \cdot \left[\begin{array}{l} v^n - \left(v - M\sqrt{\frac{2b}{M\delta+b}}\right)^n \\ \quad -n \cdot M\sqrt{\frac{2b}{M\delta+b}} \cdot \left(v - M\sqrt{\frac{2b}{M\delta+b}}\right)^{n-1} \end{array}\right] \tag{3.33}$$

It is seen again that the obtained distribution is not Poisson. For M-CSMA/CD-RC the approximations depend on M values. In case of $M = 1$, the mean value of Poisson distribution is $v + 2 \cdot \sqrt{M\delta} + 1.3/M$; for $2 \leq M \leq 5$, the mean becomes $v + 2 \cdot \sqrt{M\delta} + 1.3/M$; for $M = 6$, it is $v + 2 \cdot \sqrt{M\delta} + 1/M$. Finally, for $M \geq 7$ the mean is $v + \sqrt{M\delta} + 1/M$. The approximate Poisson distributions are:

$$Pr\{n \mid C\} = \frac{\left(v + 2 \cdot \sqrt{M\delta} + 1.3/M\right)^n \cdot \exp\left(-v - 2 \cdot \sqrt{M\delta} - 1.3/M\right)}{n!}, \qquad M = 1, \tag{3.34}$$

$$Pr\{n \mid C\} = \frac{\left(v + 2 \cdot \sqrt{M\delta} + 1.5/M\right)^n \cdot \exp\left(-v - 2 \cdot \sqrt{M\delta} - 1.5/M\right)}{n!}, \; 2 \leq M \leq 5, \tag{3.35}$$

$$Pr\{n \mid C\} = \frac{\left(v + 2 \cdot \sqrt{M\delta} + 1/M\right)^n \cdot \exp\left(-v - 2 \cdot \sqrt{M\delta} - 1/M\right)}{n!}, \qquad M = 6, \tag{3.36}$$

$$Pr\{n \mid C\} = \frac{\left(v + \sqrt{M\delta} + 1/M\right)^n \cdot \exp\left(-v - \sqrt{M\delta} - 1/M\right)}{n!}, \qquad M \geq 7, \tag{3.37}$$

Figure 3.6 is the plot of actual and approximating Poisson distributions for $M = 1$, Figure 3.7 for $M = 5$, Figure 3.8 for $M = 6$, and Figure 3.9 for $M = 10$. The figures show that the optimized approximations especially for increasing v values are pretty good.

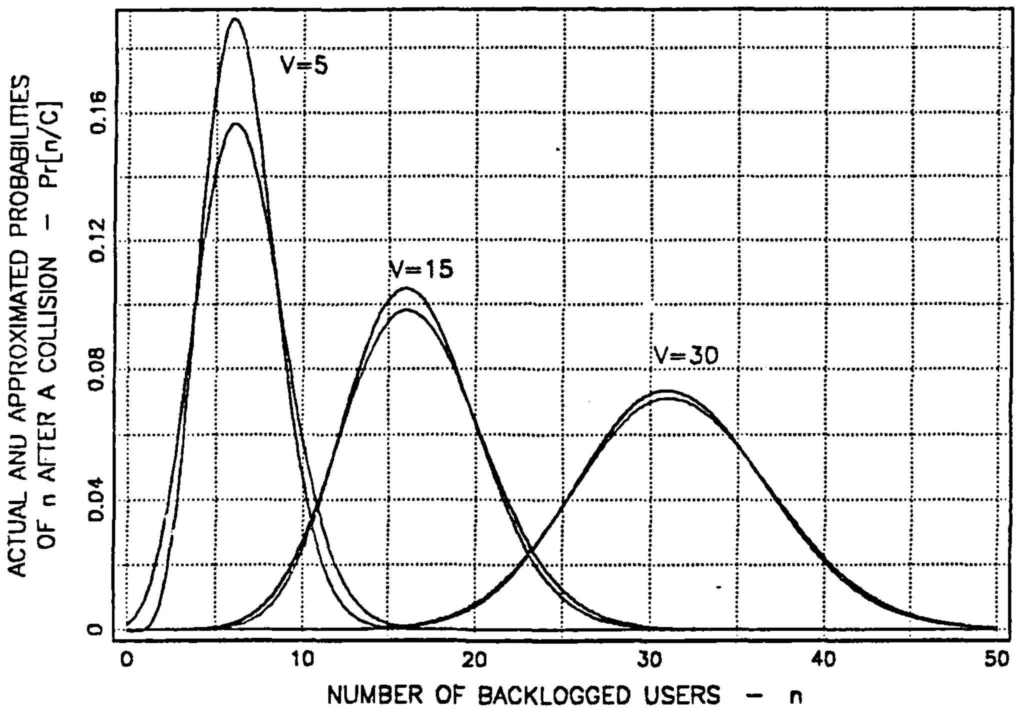

Figure 3.6: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a M–CSMA/CD–RC Network for $M = 1$.

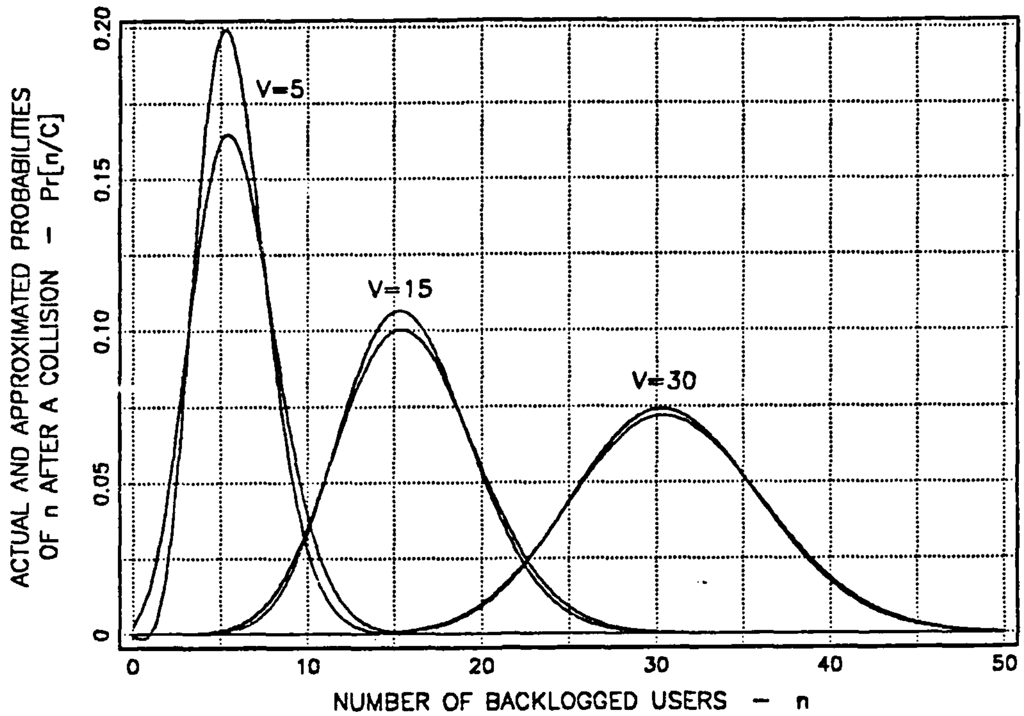

Figure 3.7: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a M–CSMA/CD–RC Network for $M = 5$.

The final step is the evaluation of the updated estimate v_{k+1} for pseudo-Bayesian M–CSMA/CD–RC according to the following:

$$v_{k+1} = \begin{cases} v_k \cdot (1 - q_i) + \lambda b; & \text{for idle} \\ v_k \cdot (1 - q_i) + \lambda(1 + b); & \text{for success} \\ v_k + 2\sqrt{M\delta} + 1.3/M + \lambda(1 + b); & \text{for collision } M = 1 \\ v_k + 2\sqrt{M\delta} + 1.5/M + \lambda(1 + b); & \text{for collision } 2 \le M \le 5 \\ v_k + 2\sqrt{M\delta} + 1/M + \lambda(1 + b); & \text{for collision } M = 6 \\ v_k + \sqrt{M\delta} + 1/M + \lambda(1 + b); & \text{for collision } M \ge 7 \end{cases} \tag{3.38}$$

C. M–CSMA–IC AND M–CSMA/CD–IC

A drawback of the random choice (RC) procedure is that packets may be rescheduled while some channels are idle. To improve performance the stations are restricted to choose at random only between those channels that are sensed idle.

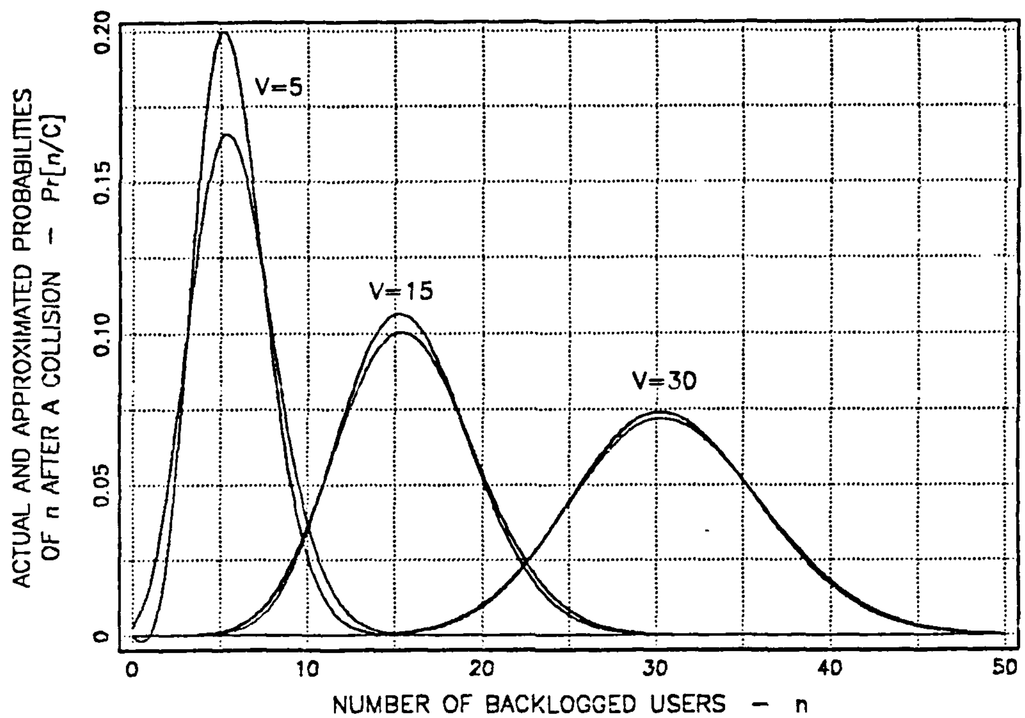

Figure 3.8: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a M–CSMA/CD–RC Network for $M = 6$.

In this way packets are rescheduled only when all channels are busy. For simplicity it is again assumed that when M channels are sensed idle, each one can be chosen with probability $1/M$.

Defining P_1 to be the probability that at least one channel is sensed idle; $P_{1,i}$ to be the probability that the ith channel is sensed idle; P_{nci} to be the probability that a packet transmitted on channel i does not collide, the probability that a packet is successfully transmitted on channel is then obtained

$$P_{si} = P_{1,i} \cdot P_{nci} \tag{3.39}$$

The system throughput is again evaluated using Equation 3.7, where now

$$P_s = P_1 \cdot P_{nc} \tag{3.40}$$

P_{nc} is the probability of no collision on the chosen channel. The channels are

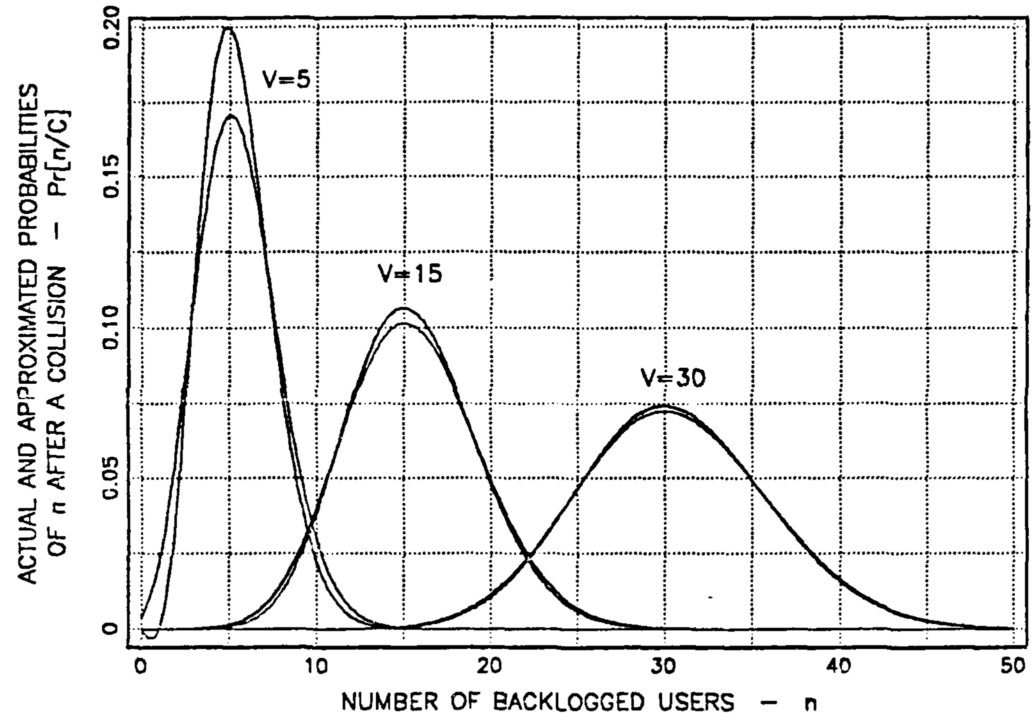

Figure 3.9: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a M–CSMA/CD–RC Network for $M = 10$.

assumed independent, that is to say that the state of one channel does not depend on the state of other channels. This is an approximation because of the procedure followed in the channel selection.

The traffic offered to each channel in the system is obviously not Poisson, thus the exact analysis of this protocol is rather difficult. In order to obtain an estimate of the protocol performance, the traffic offered to the ith channel is assumed Poisson. Channels are randomly chosen only after the carrier sensing operation so that the total offered traffic is partitioned among those channel that are sensed idle; thus

$$g_i = g \cdot T_i/T_0 \cdot Pr\{i \text{ chosen } \mid i \text{ sensed idle}\} \tag{3.41}$$

Assuming that a channel is idle independently of the other channel states, condi-

tioning on the number of idle channels and averaging

$$\begin{aligned} g_i &= g \cdot M \cdot \sum_{j=0}^{M-1} \frac{1}{j+1} \cdot c(M-1,j) \cdot (1-P_{1,i})^{M-1-j} \cdot P_{1,i}^{j} \\ &= g \cdot \frac{1-(1-P_{1,i})^M}{P_{1,i}} \end{aligned} \tag{3.42}$$

where $P_{1,i}$ is the probability that a station senses the chosen channel idle for M–CSMA–IC is

$$\begin{aligned} P_{1,i} &= \frac{\overline{I}+b_i}{\overline{I}+\overline{B}} \\ &= \frac{b_i}{1+b_i-\exp(-b_i \cdot g_i)} \end{aligned} \tag{3.43}$$

where $\overline{I}$ and $\overline{B}$ denote the average length in time of the idle period and the busy period respectively. Then the probability, that a packet transmitted on channel i does not collide, can be expressed as

$$P_{nci} = \frac{P_s}{\overline{I}+b_i} = \exp(-b_i \cdot g_i) \tag{3.44}$$

Since the offered traffic to all channels in steady state is the same,

$$P_1 = 1-(1-P_{1,i})^M \tag{3.45}$$

and

$$P_{nc} = \sum_{i=1}^{M} P_{nci} \cdot P_i \tag{3.46}$$

where P_i is the probability of choosing the ith channel. In steady state i. 's assumed $P_i = 1/M, b_i = b/M$ and thus $P_{nc} = P_{nci}$. Finally, substituting into Equation 3.7

$$S_i = g_i \cdot P_{1,i} \cdot P_{nci} \tag{3.47}$$

and the throughput expression for M–CSMA–IC is obtained as

$$S_i = \frac{b \cdot g_i \cdot \exp(-b \cdot g_i/M)}{b+M(1-\exp(-b \cdot g_i/M))} \tag{3.48}$$

Defining the symbol G_i as representing $b \cdot g_i$, Equation 3.48 becomes

$$S_i = \frac{G_i \cdot \exp(-G_i/M)}{b+M(1-\exp(-G_i/M))} \tag{3.49}$$

The system throughput for M–CSMA/CD–IC is evaluated using the same procedure from M–CSMA–IC, where now the probability of sensing ith channel idle is

$$P_{1,i} = \frac{b_i}{b_i + \delta\left(-\exp(-b_i \cdot g_i) - b_i \cdot g_i \cdot \exp(-b_i \cdot g_i)\right) + b_i \cdot g_i \cdot \exp(-b_i \cdot g_i)} \quad (3.50)$$

and the probability that the packet transmitted on channel i does not collide is

$$P_{nci} = \exp(-b_i \cdot g_i) \quad (3.51)$$

The throughput expression for M–CSMA/CD–IC is from Equation 3.47 and for $b_i = b/M$,

$$S_i = \frac{b \cdot g_i \cdot \exp(-b \cdot g_i/M)}{b + M \cdot \delta\left(1 - \exp\left(\frac{-b \cdot g_i}{M}\right) - \frac{b \cdot g_i}{M} \cdot \exp\left(\frac{-b \cdot g_i}{M}\right)\right) + b \cdot g_i \cdot \exp\left(\frac{-b \cdot g_i}{M}\right)} \quad (3.52)$$

Setting $b \cdot g_i$ equal to G_i

$$S_i = \frac{G_i \cdot \exp(-G_i/M)}{b + M \cdot \delta\left(1 - \exp(-G_i/M) - G_i/M \cdot \exp(-G_i/M)\right) + G_i \cdot \exp(-G_i/M)} \quad (3.53)$$

The broadcast probabilities, which optimize the throughput expression, are for the M–CSMA–IC

$$q_i = \min\{\frac{M}{v}\sqrt{\frac{2b}{M+b}}, 1\} \quad (3.54)$$

and for M–CSMA/CD–IC

$$q_i = \min\{\frac{M}{v}\sqrt{\frac{2b}{M\delta+b}}, 1\} \quad (3.55)$$

The stabilization algorithm is again pseudo-Bayesian. As the throughput expressions for RC and IC are compared, it is seen that they are similar but the throughput expression for IC-protocol includes the subscript i, which represents the conditioning on the number of idle channels. Thus, the pseudo-Bayesian algorithm for stabilization gives the same probabilities but makes use of the available

channels which are idle and does not include the busy channels. Then, using the subscript i and Equations 3.31 and 3.32, the probability distributions of n, given an idle or given a success, are obtained for slotted M-CSMA/CD-IC,

$$Pr\{n \mid I_i\} = \frac{\left(v - M\sqrt{\frac{2b}{M\delta+b}}\right)^n \cdot \exp\left(-v + M\sqrt{\frac{2b}{M\delta+b}}\right)}{n!} \tag{3.56}$$

and

$$Pr\{n \mid S_i\} = \frac{\left(v - M\sqrt{\frac{2b}{M\delta+b}}\right)^{n-1} \cdot \exp\left(-v + M\sqrt{\frac{2b}{M\delta+b}}\right)}{(n-1)!} \tag{3.57}$$

Finally, the probability distribution of n given a collision is

$$\begin{aligned} Pr\{n \mid C_i\} \;=\; & \frac{\exp\left(-v + M\sqrt{\frac{2b}{M\delta+b}}\right)}{n! \cdot \left[\exp\left(M\sqrt{\frac{2b}{M\delta+b}}\right) - 1 - M\sqrt{\frac{2b}{M\delta+b}}\right]} \\ & \cdot \begin{bmatrix} v^n - \left(v - M\sqrt{\frac{2b}{M\delta+b}}\right)^n \\ \qquad -n \cdot M \cdot \sqrt{\frac{2b}{M\delta+b}}\left(v - M\sqrt{\frac{2b}{M\delta+b}}\right)^{n-1} \end{bmatrix} \end{aligned} \tag{3.58}$$

which is not a Poisson distribution and can be approximated by the following Poisson distributions for the given specific M values. The approximating probability distributions are

$$Pr\{n \mid C_i\} = \frac{(v + 2\sqrt{M\delta} + 1.3/M)^n \cdot \exp(-v - 2\sqrt{M\delta} - 1.3/M)}{n!}, \quad M = 1 \tag{3.59}$$

$$Pr\{n \mid C_i\} = \frac{(v + 2\sqrt{M\delta} + 1.5/M)^n \cdot \exp(-v - 2\sqrt{M\delta} - 1.5/M)}{n!}, \quad 2 \le M \le 5; \tag{3.60}$$

$$Pr\{n \mid C_i\} = \frac{(v + 2\sqrt{M\delta} + 1/M)^n \cdot \exp(-v - 2\sqrt{M\delta} - 1/M)}{n!}, \quad M = 6 \tag{3.61}$$

$$Pr\{n \mid C_i\} = \frac{(v + \sqrt{M\delta} + 1/M)^n \cdot \exp(-v - \sqrt{M\delta} - 1/M)}{n!}, \quad M \ge 7 \tag{3.62}$$

Figure 3.10 is the plot of actual and approximating Poisson distributions for $M = 1$, Figure 3.11 for $M = 5$, Figure 3.12 for $M = 6$, and Figure 3.13 for $M = 10$, which show that for M-CSMA/CD-IC the approximations are very close, especially for increasing v values.

The last step is the determination of updated estimates which are

$$v_{k,i+1} = \begin{cases} v \cdot (1 - q_i) + \lambda b; & \text{for idle} \\ v \cdot (1 - q_i) + \lambda(1 + b); & \text{for success} \\ v + 2\sqrt{M\delta} + 1.3/M + \lambda(1 + b); & \text{for collision } M = 1 \\ v + 2\sqrt{M\delta} + 1.5/M + \lambda(1 + b); & \text{for collision } 2 \leq M \leq 5 \\ v + 2\sqrt{M\delta} + 1/M + \lambda(1 + b); & \text{for collision } M = 6 \\ v + \sqrt{M\delta} + 1/M + \lambda(1 + b); & \text{for collision } M \geq 7 \end{cases} \tag{3.63}$$

where q_i is given in Equation 3.55.

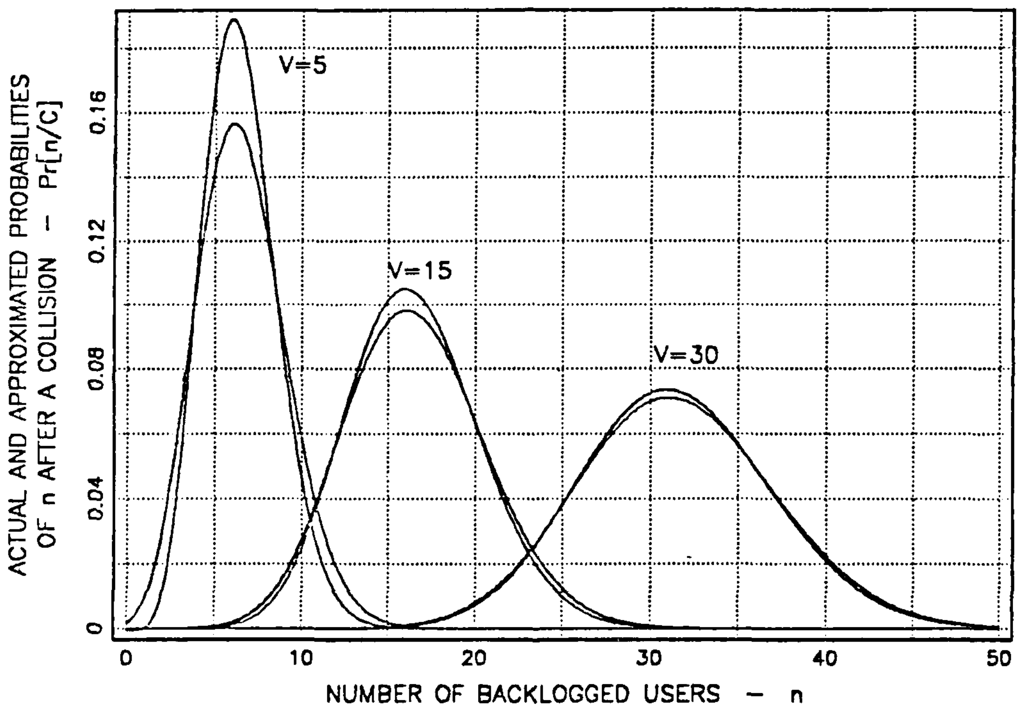

Figure 3.10: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a Slotted M–CSMA/CD–IC Network for $M = 1$.

The slotted CSMA , where the value of δ in CSMA/CD equal to one, is a case that makes the M–CSMA–IC calculation easier. For M–CSMA–IC the probability

distribution of n given an idle is from Equation 3.56

$$Pr\{n \mid I_i\} = \frac{\left(v - M\sqrt{\frac{2b}{M+b}}\right)^n \cdot \exp\left(-v + M\sqrt{\frac{2b}{M+b}}\right)}{n!} \tag{3.64}$$

and the probability distribution of n given a success is using Equation 3.57

$$Pr\{n \mid S_i\} = \frac{\left(v - M\sqrt{\frac{2b}{M+b}}\right)^{n-1} \cdot \exp\left(-v + M\sqrt{\frac{2b}{M+b}}\right)}{(n-1)!} \tag{3.65}$$

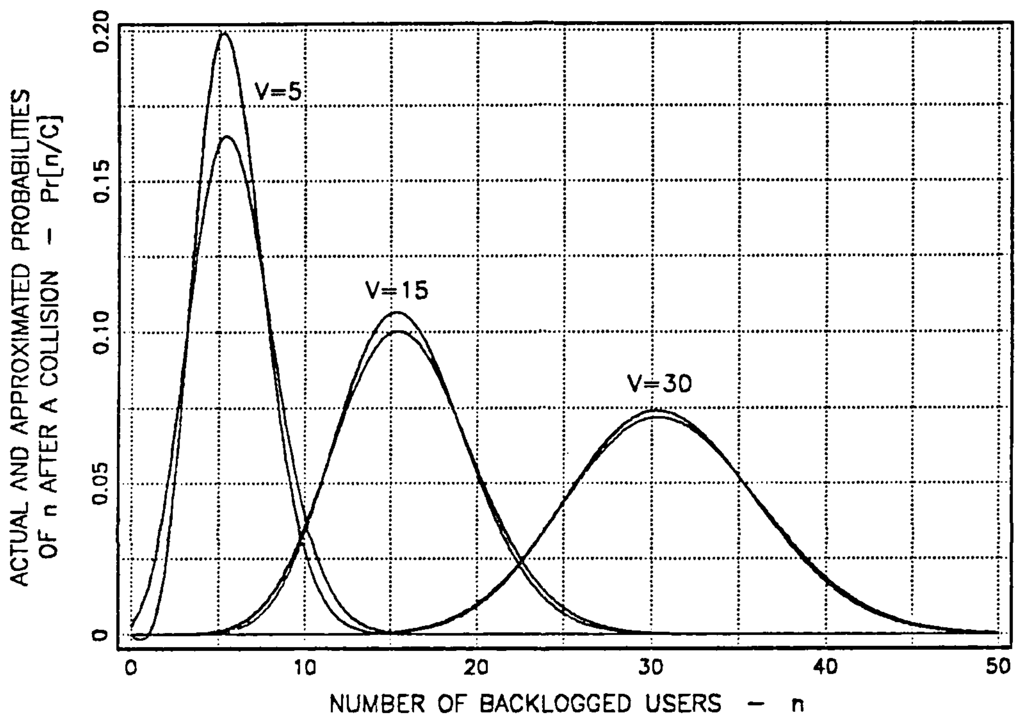

Figure 3.11: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a Slotted M–CSMA/CD–IC Network for $M = 5$.

The probability distribution of n given a collision is obtained from Equation 3.58 for M–CSMA–IC

$$\begin{aligned} Pr\{n \mid C_i\} \quad = \quad & \frac{\exp\left(-v + M\sqrt{\frac{2b}{M+b}}\right)}{n! \cdot \left[\exp\left(M\sqrt{\frac{2b}{M+b}}\right) - 1 - M\sqrt{\frac{2b}{M+b}}\right]} \\ & \cdot \begin{bmatrix} v^n - \left(v - M\sqrt{\frac{2b}{M+b}}\right)^n \\ \qquad - n \cdot M\sqrt{\frac{2b}{M+b}} \cdot \left(v - M\sqrt{\frac{2b}{M+b}}\right)^{n-1} \end{bmatrix} \end{aligned} \tag{3.66}$$

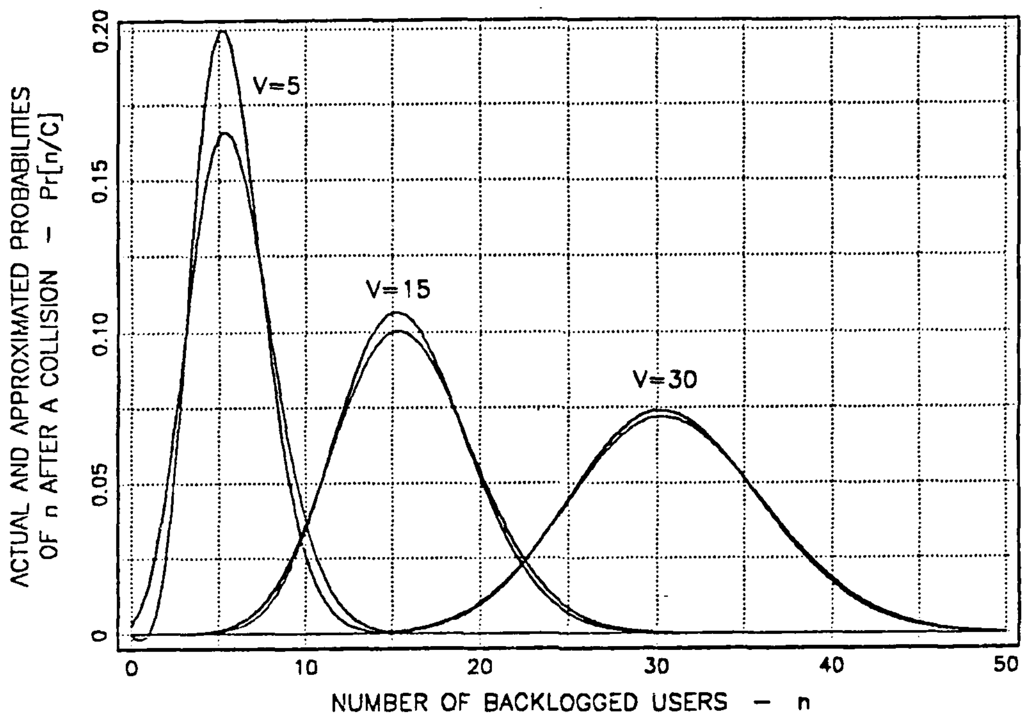

Figure 3.12: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a Slotted M–CSMA/CD–IC Network for $M = 6$.

The Poisson approximating distribution of n given a collision for $M = 1$ is

$$Pr\{n \mid C_i\} = \frac{(v + 2\sqrt{M})^n \cdot \exp(-v - 2\sqrt{M})}{n!} \tag{3.67}$$

and for $M > 1$

$$Pr\{n \mid C_i\} = \frac{\left[v + 2(\sqrt{M} - 0.25)\right]^n \cdot \exp\left(-v - 2(\sqrt{M} - 0.25M)\right)}{n!} \tag{3.68}$$

Figure 3.14 shows the actual and approximating distributions for $M = 1$, Figure 3.15 for $M = 5$ and Figure 3.16 for $M = 10$.

Updating the estimate v of the number of backlogged stations for M-CSMA-IC

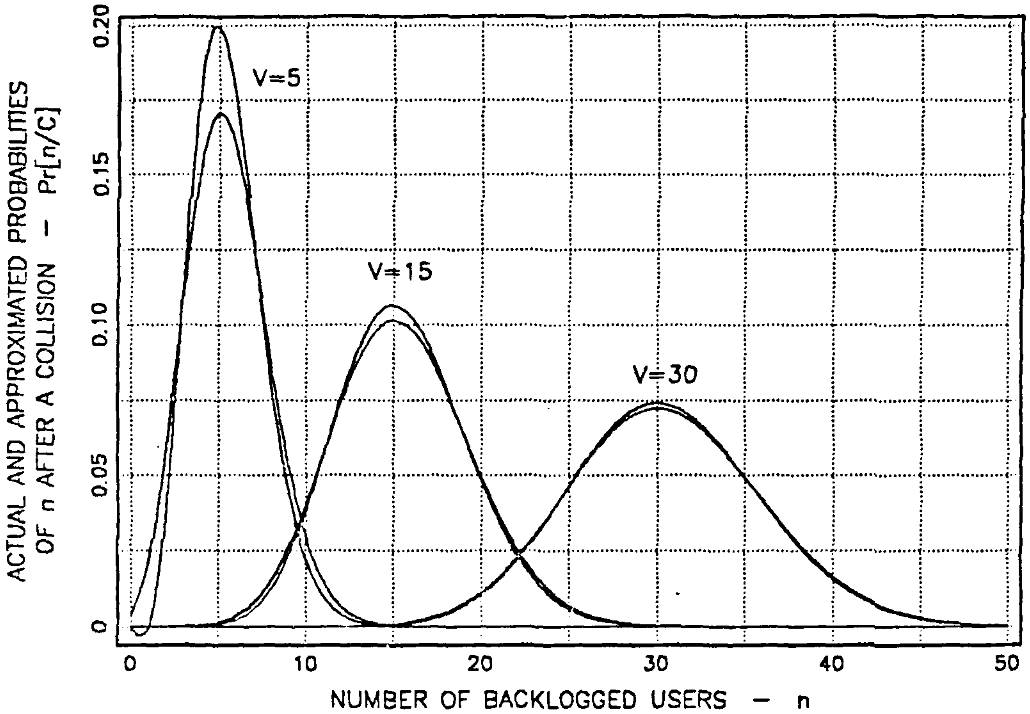

Figure 3.13: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a Slotted M–CSMA/CD–IC Network for $M = 10$.

case, the obtained updated estimate is

$$v_{k,i+1} = \begin{cases} v \cdot (1 - q_i) + \lambda b; & \text{for idle} \\ v \cdot (1 - q_i) + \lambda(1 + b); & \text{for success} \\ v + 2\sqrt{M} + \lambda(1 + b); & \text{for collision } M = 1 \\ v + 2(\sqrt{M} - 0.25M) + \lambda(1 + b); & \text{for collision } M > 1 \end{cases} \tag{3.69}$$

where q_i is given in Equation 3.54.

The pseudo-Bayesian slotted M-CSMA-IC, M–CSMA/CD–IC are similar in behavior to pseudo-Bayesian slotted M–CSMA–RC and M–CSMA/CD–RC. In slotted M–CSMA–RC and M–CSMA/CD–RC protocols each of the backlogged packets is independently transmitted with the packet broadcast probability q_i on channel i which is randomly chosen. For slotted M–CSMA–IC and M–CSMA/CD–IC protocols each of the backlogged packets is independently transmitted with the packet

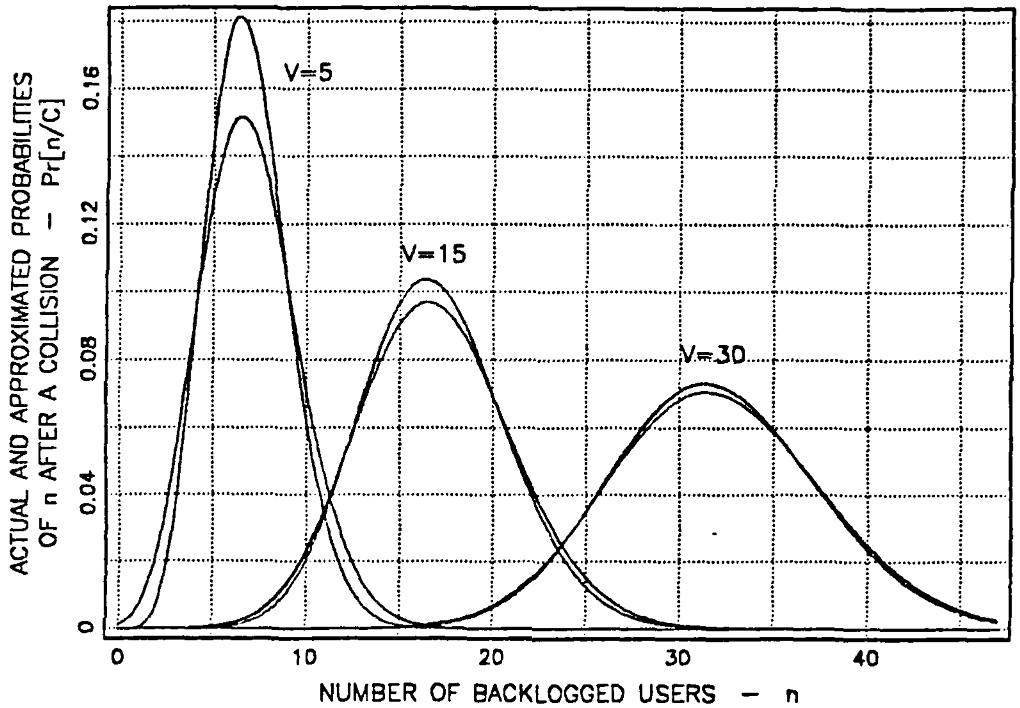

Figure 3.14: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a Slotted M–CSMA–IC Network for $M = 1$.

broadcast probability q_i on the idle sensed channel. The traffic offered to the ith channel in both RC and IC protocols is Poisson with rate G_i. The number of backlogged stations n and the user's estimate of the number of backlogged stations v characterize the system. For large backlogs the channel traffic rate G_i is one packet per slot and the throughput of the system is maximized for given q_i values in Equations 3.17, 3.29, 3.54, and 3.55. Considering $n > v$ for the chosen channel, the traffic rate will be larger than one packet per slot which means more frequent collision slots from idle or success slots. In this case v grows faster than n, $n - v$ converges to zero and throughput approaches the maximum achievable. In case of $n < v$, where the ith channel traffic rate will be smaller than one packet per slot, idle and success slots will occur more frequently. The reduction in the estimate due

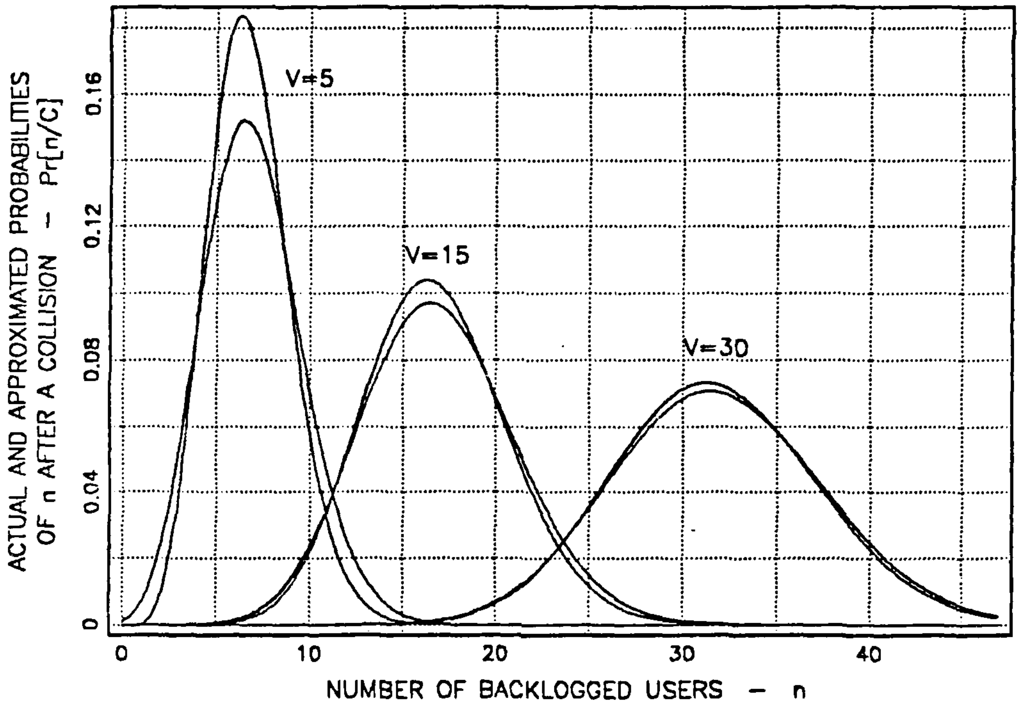

Figure 3.15: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a Slotted M–CSMA–IC Network for $M = 5$.

to idle slots will cause v to decrease more rapidly than n, the difference $n - v$ will converge to zero and throughput will increase.

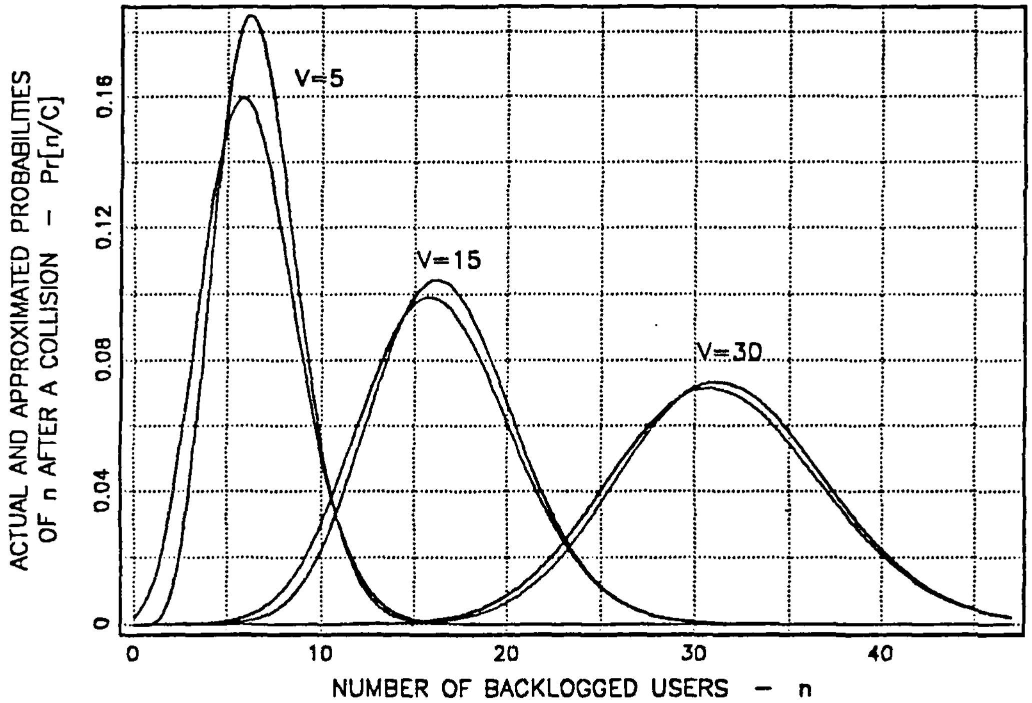

Figure 3.16: Comparison of the Actual and Poisson Approximating Probability Distributions of the Number n of Backlogged Users After a Collision Occurs in a Slotted M–CSMA–IC Network for $M = 10$.

IV. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The theory needed to analyze the stability of slotted CSMA and slotted CSMA/CD networks using pseudo-Bayesian algorithm has been developed. The pseudo-Bayesian algorithm is exceptionally effective in practice, since it makes the best possible use of the information available on the network in determining the broadcast probabilities to use. [Ref. 5] The approximated probability distributions of n given that a collision occurs in the previous slot matched very well with the actual distributions which are derived from pseudo-Bayesian algorithm. The stabilization of slotted CSMA and slotted CSMA/CD protocols using two power levels performed in a similar manner such as in one power level. In the model considered for two power levels, priority classes among the network users are avoided since each station chooses a received power level at random from a given set for each data packet transmitted.

M–LAN's are a concept that can be exploited in the design of high performance packet switching broadcast networks for local area applications. They also provide a method to upgrade existing networks by adding new equipment instead of replacing old equipment. [Ref. 7: p. 378] In this thesis the stabilization of multiple access protocols derived from slotted CSMA and slotted CSMA/CD protocols utilizing the pseudo-Bayesian technique was developed and it provided improvements with respect to the single channel protocol.

B. RECOMMENDATIONS

The pseudo-Bayesian stabilization for unslotted CSMA and unslotted CSMA/CD should perform well in maintaining the channel traffic rate that yield the maximum throughput achievable which should be confirmed. The effects of fading on the pseudo-Bayesian algorithm should also be studied. Analyzing M–CSMA and M–CSMA/CD with random power levels topics such as the optimum number of power levels and stabilization should be carried out.

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